Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.1-Sine/67-4.1.1.1-a+b-sin-^n

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [72]. This is test number [67].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (72)	0.00 (0)
Mathematica	100.00 (72)	0.00 (0)
Maple	65.28 (47)	34.72 (25)
Fricas	65.28 (47)	34.72 (25)
Giac	54.17 (39)	45.83 (33)
Mupad	50.00 (36)	50.00 (36)
Maxima	44.44 (32)	55.56 (40)
Sympy	44.44 (32)	55.56 (40)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

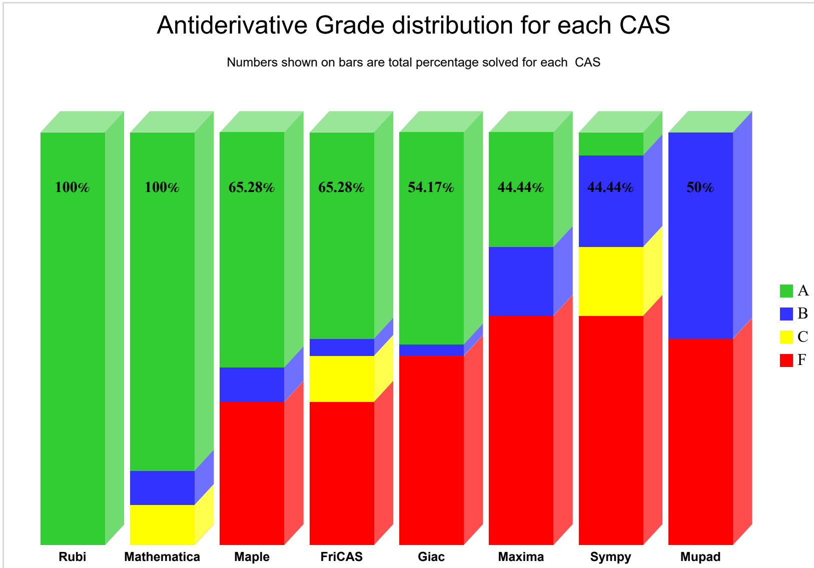
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

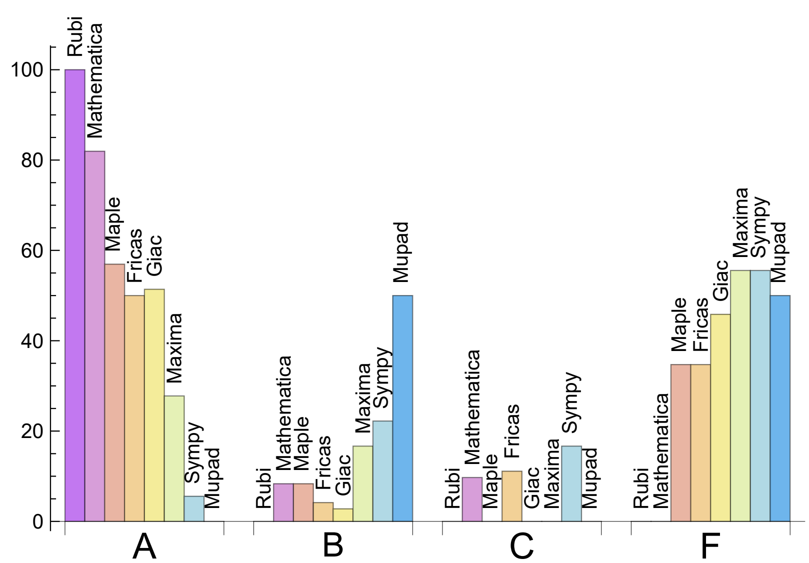
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	81.944	8.333	9.722	0.000
Maple	56.944	8.333	0.000	34.722
Giac	51.389	2.778	0.000	45.833
Fricas	50.000	4.167	11.111	34.722
Maxima	27.778	16.667	0.000	55.556
Sympy	5.556	22.222	16.667	55.556
Mupad	0.000	50.000	0.000	50.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	25	100.00	0.00	0.00
Maple	25	100.00	0.00	0.00
Giac	33	100.00	0.00	0.00
Mupad	36	0.00	100.00	0.00
Maxima	40	100.00	0.00	0.00
Sympy	40	95.00	5.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Fricas	0.25
Giac	0.31
Maple	0.32
Rubi	0.35
Mathematica	0.36
Sympy	1.65
Mupad	4.71

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	92.00	0.99	74.00	1.00
Mupad	92.89	1.12	83.00	1.21
Giac	97.90	1.23	107.00	1.37
Mathematica	124.12	1.42	115.00	1.38
Maxima	147.38	1.58	144.00	1.68
Maple	162.09	1.35	91.00	1.08
Fricas	175.94	1.59	130.00	1.18
Sympy	891.34	8.61	691.50	8.79

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

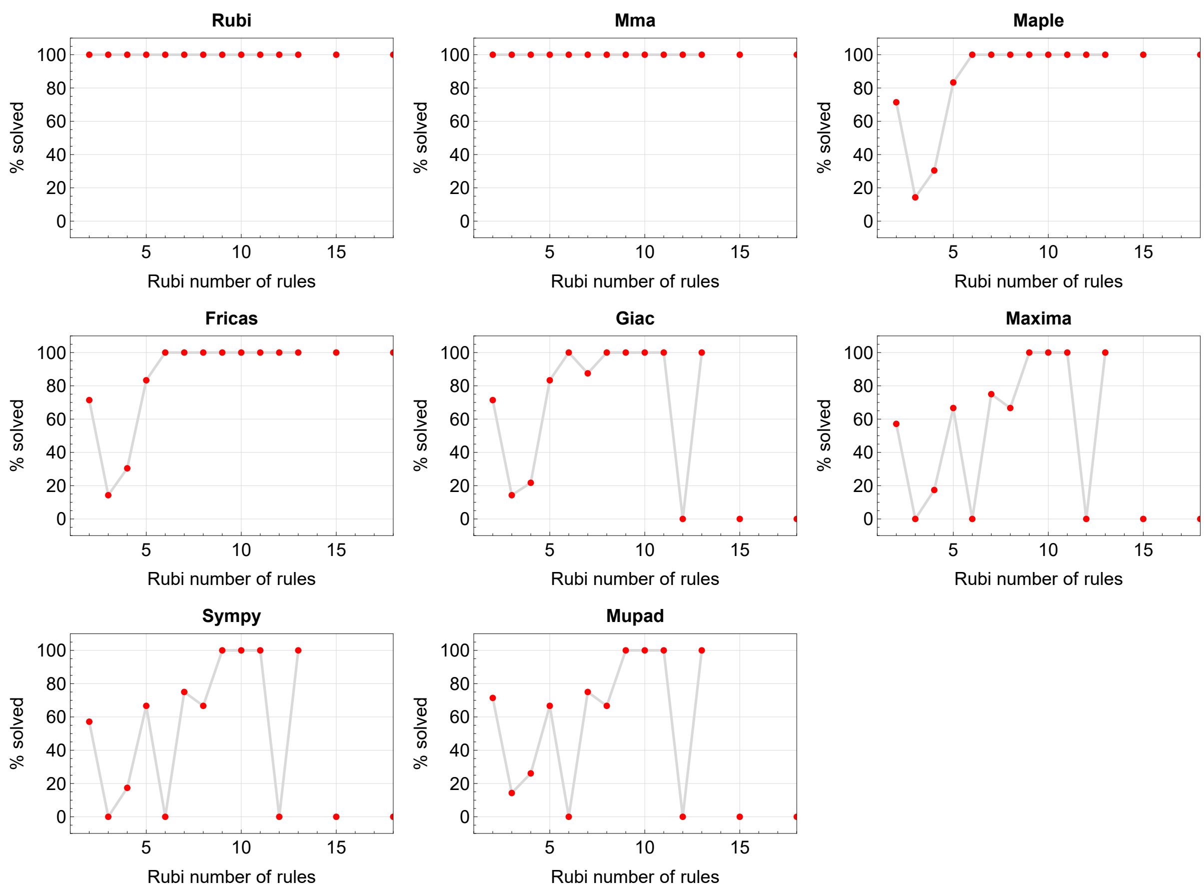


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

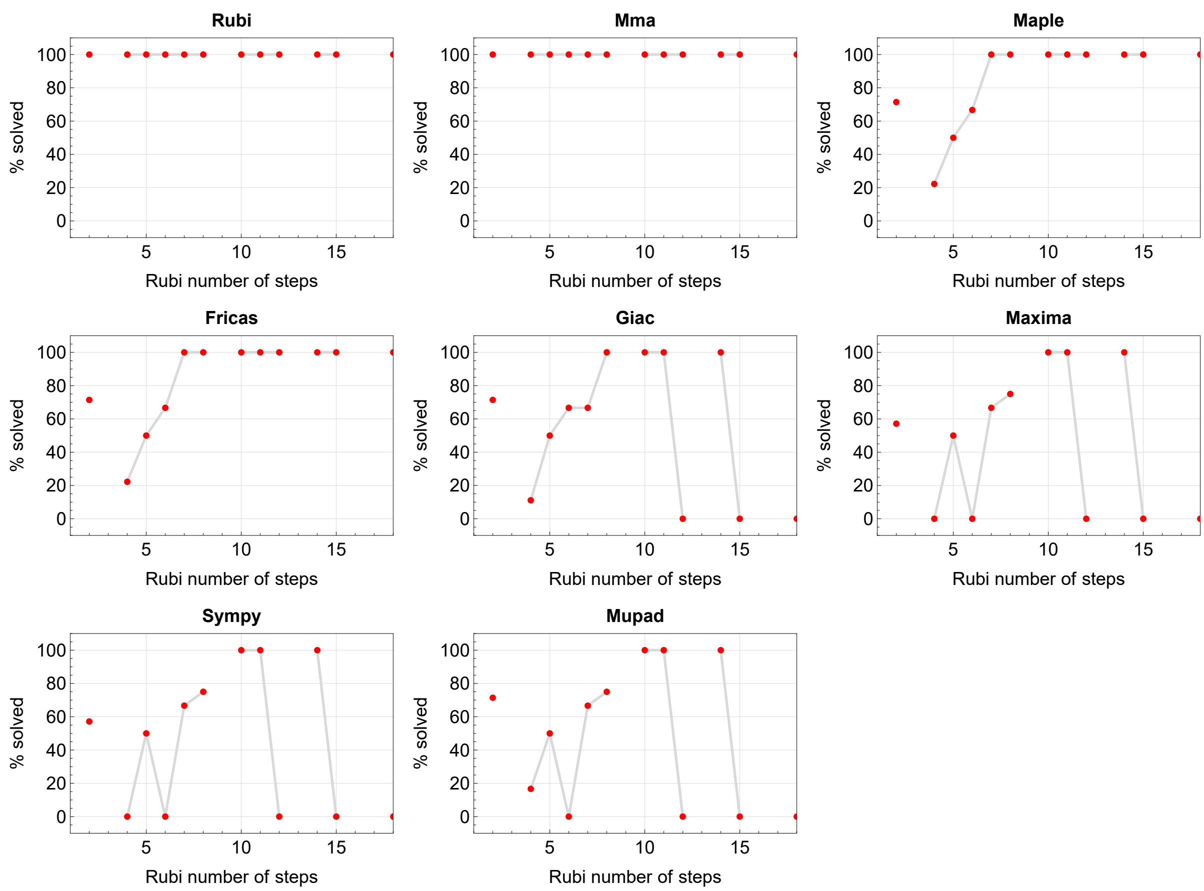


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

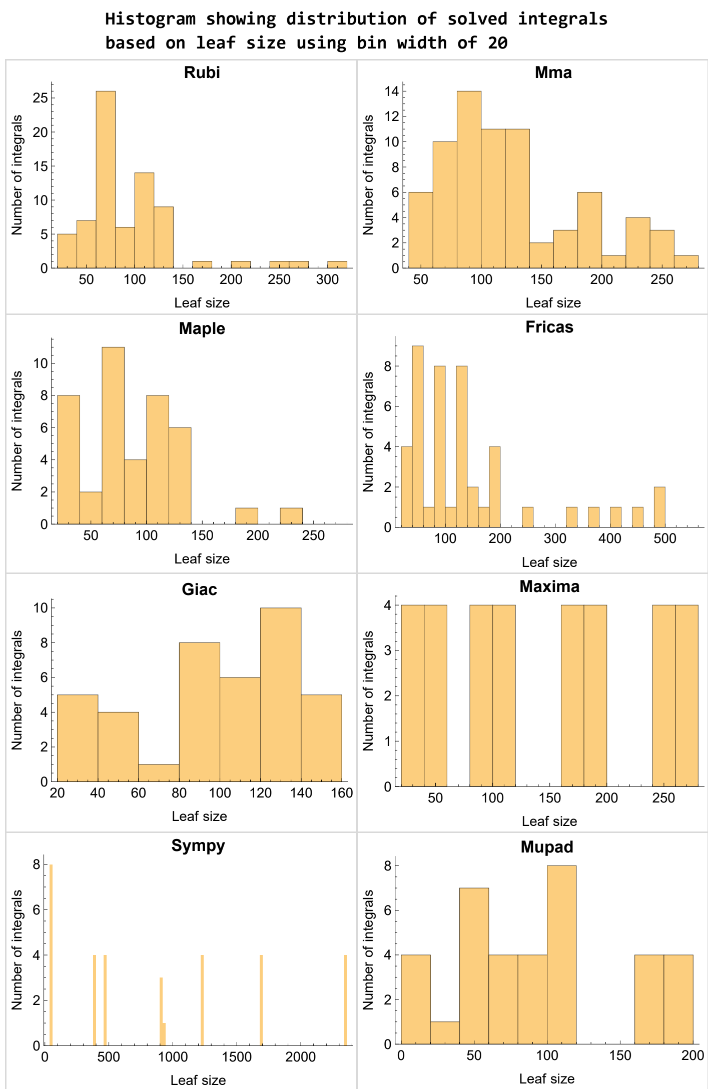


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

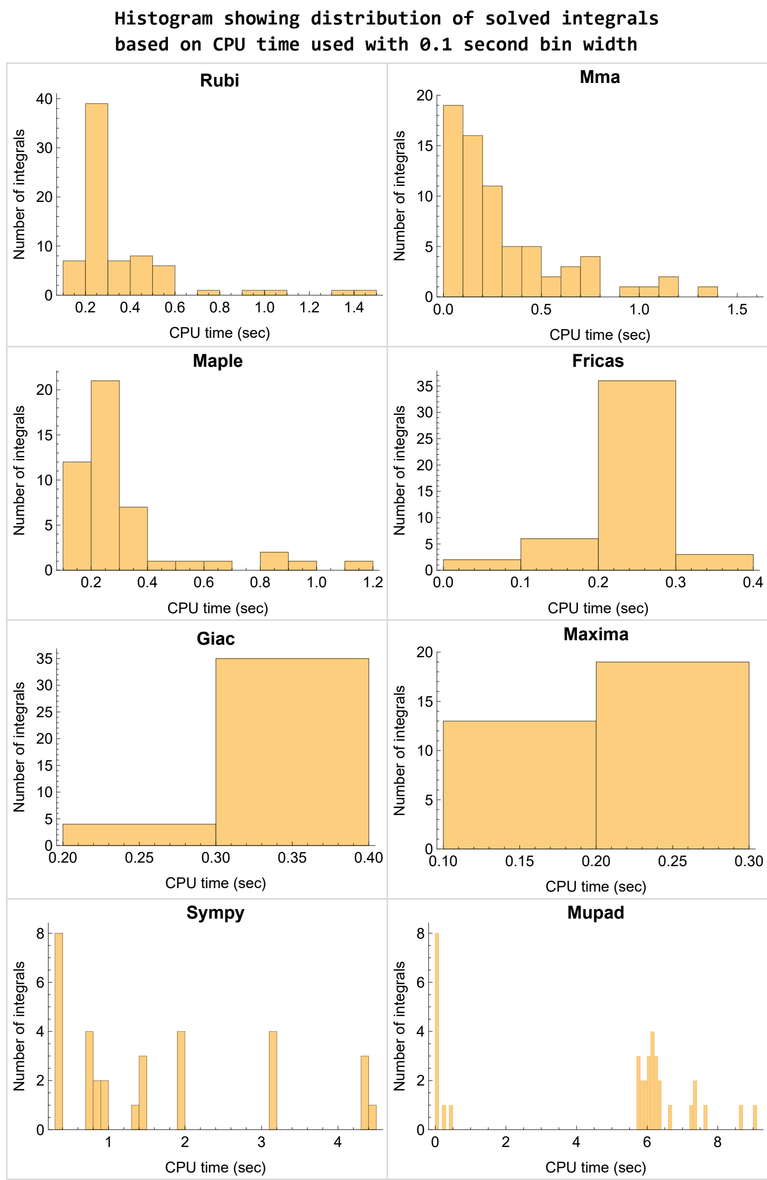


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

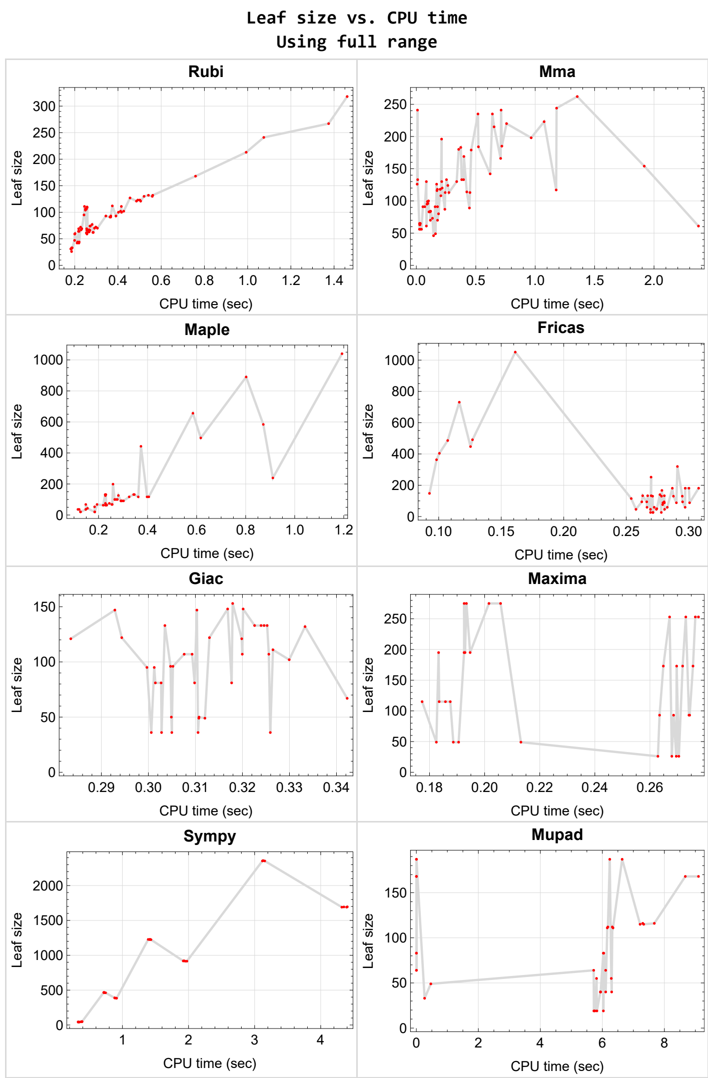


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

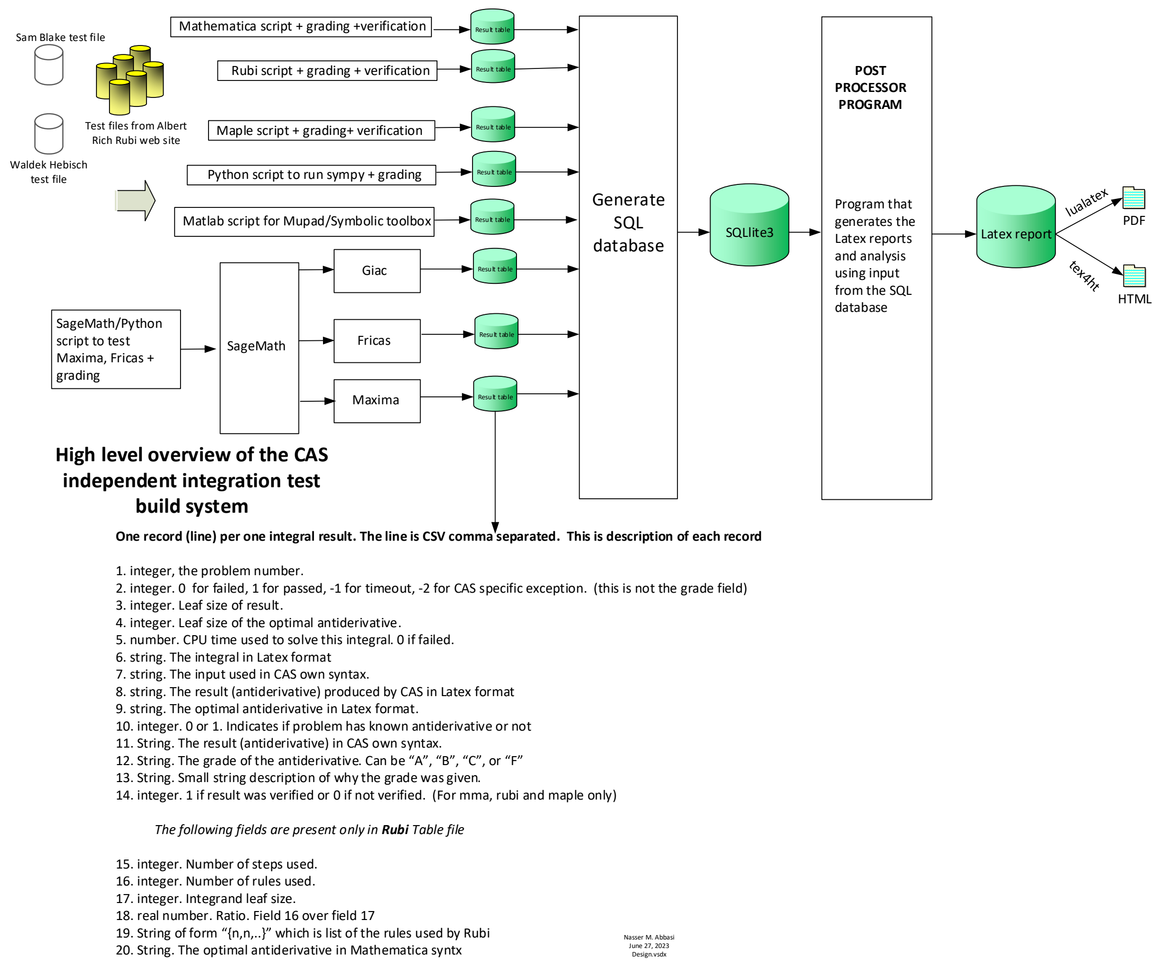
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 9, 11, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

B grade { 4, 8, 10, 12, 58, 63 }

C grade { 5, 6, 7, 14, 15, 16, 17 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 56, 57 }

B grade { 7, 50, 51, 52, 53, 55 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 5, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade { 4, 6, 7 }

C grade { 50, 51, 52, 53, 54, 55, 56, 57 }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 18, 19, 22, 23, 26, 27, 30, 31, 34, 35, 36, 38, 39, 40, 42, 43, 44, 46, 47, 48 }

B grade { 20, 21, 24, 25, 28, 29, 32, 33, 37, 41, 45, 49 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 7, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade { 5, 6 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 4, 5, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 34, 38, 42, 46 }

B grade { 18, 22, 26, 30, 35, 36, 37, 39, 40, 41, 43, 44, 45, 47, 48, 49 }

C grade { 19, 20, 21, 23, 24, 25, 27, 28, 29, 31, 32, 33 }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { 1, 50 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	127	154	75	0	140	0	132	0
N.S.	1	1.07	1.29	0.63	0.00	1.18	0.00	1.11	0.00
time (sec)	N/A	0.447	1.918	0.244	0.000	0.277	0.000	0.333	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	117	65	0	115	0	102	0
N.S.	1	1.04	1.31	0.73	0.00	1.29	0.00	1.15	0.00
time (sec)	N/A	0.350	1.177	0.233	0.000	0.254	0.000	0.330	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	89	53	0	76	0	67	0
N.S.	1	1.00	1.51	0.90	0.00	1.29	0.00	1.14	0.00
time (sec)	N/A	0.257	0.447	0.184	0.000	0.279	0.000	0.342	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	65	43	0	50	0	36	33
N.S.	1	1.00	2.50	1.65	0.00	1.92	0.00	1.38	1.27
time (sec)	N/A	0.179	0.033	0.155	0.000	0.274	0.000	0.326	0.266

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	75	0	167	0	111	49
N.S.	1	1.00	1.55	1.60	0.00	3.55	0.00	2.36	1.04
time (sec)	N/A	0.201	0.135	0.228	0.000	0.279	0.000	0.326	0.467

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	108	125	0	252	0	133	0
N.S.	1	1.00	1.40	1.62	0.00	3.27	0.00	1.73	0.00
time (sec)	N/A	0.274	0.206	0.228	0.000	0.270	0.000	0.325	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	196	199	0	320	0	153	0
N.S.	1	1.05	1.83	1.86	0.00	2.99	0.00	1.43	0.00
time (sec)	N/A	0.364	0.212	0.259	0.000	0.291	0.000	0.318	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	223	0	0	0	0	0	0
N.S.	1	1.00	3.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	1.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	124	0	0	0	0	0	0
N.S.	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.265	0.000	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	169	0	0	0	0	0	0
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.400	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	70	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	215	0	0	0	0	0	0
N.S.	1	1.00	3.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.654	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	130	0	0	0	0	0	0
N.S.	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.340	0.000	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	80	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.187	0.000	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	46	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	56	20	26	26	46	49	40
N.S.	1	1.00	1.81	0.65	0.84	0.84	1.48	1.58	1.29
time (sec)	N/A	0.182	0.043	0.184	0.268	0.271	0.373	0.312	6.107

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	91	63	93	59	388	95	83
N.S.	1	1.09	1.62	1.12	1.66	1.05	6.93	1.70	1.48
time (sec)	N/A	0.248	0.158	0.231	0.269	0.297	0.883	0.301	6.031

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	91	113	91	173	94	918	121	111
N.S.	1	1.12	1.40	1.12	2.14	1.16	11.33	1.49	1.37
time (sec)	N/A	0.353	0.274	0.302	0.270	0.280	1.920	0.320	6.158

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	121	133	117	253	130	1693	147	187
N.S.	1	1.14	1.25	1.10	2.39	1.23	15.97	1.39	1.76
time (sec)	N/A	0.491	0.382	0.398	0.276	0.295	4.351	0.310	6.238

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	56	20	26	26	46	50	40
N.S.	1	1.00	1.70	0.61	0.79	0.79	1.39	1.52	1.21
time (sec)	N/A	0.173	0.034	0.185	0.270	0.271	0.387	0.311	6.299

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	63	91	63	93	59	384	96	83
N.S.	1	1.09	1.57	1.09	1.60	1.02	6.62	1.66	1.43
time (sec)	N/A	0.252	0.178	0.233	0.263	0.283	0.912	0.305	6.045

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	93	113	91	173	94	915	122	111
N.S.	1	1.12	1.36	1.10	2.08	1.13	11.02	1.47	1.34
time (sec)	N/A	0.362	0.451	0.296	0.276	0.266	1.976	0.313	6.343

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	123	133	117	253	130	1690	148	187
N.S.	1	1.14	1.23	1.08	2.34	1.20	15.65	1.37	1.73
time (sec)	N/A	0.478	0.397	0.405	0.273	0.278	4.393	0.317	6.638

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	56	20	26	26	48	50	40
N.S.	1	1.00	1.70	0.61	0.79	0.79	1.45	1.52	1.21
time (sec)	N/A	0.178	0.026	0.127	0.263	0.278	0.390	0.305	5.945

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	63	91	63	93	59	384	96	83
N.S.	1	1.09	1.57	1.09	1.60	1.02	6.62	1.66	1.43
time (sec)	N/A	0.253	0.055	0.219	0.274	0.267	0.906	0.305	0.002

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	93	113	91	173	94	915	122	112
N.S.	1	1.12	1.36	1.10	2.08	1.13	11.02	1.47	1.35
time (sec)	N/A	0.351	0.243	0.292	0.272	0.262	1.950	0.294	6.178

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	123	133	117	253	130	1690	148	187
N.S.	1	1.14	1.23	1.08	2.34	1.20	15.65	1.37	1.73
time (sec)	N/A	0.486	0.011	0.362	0.267	0.288	4.320	0.320	0.003

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	56	20	26	26	49	49	40
N.S.	1	1.00	1.81	0.65	0.84	0.84	1.58	1.58	1.29
time (sec)	N/A	0.177	0.030	0.126	0.270	0.269	0.388	0.311	5.935

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	91	63	93	59	389	95	83
N.S.	1	1.09	1.62	1.12	1.66	1.05	6.95	1.70	1.48
time (sec)	N/A	0.247	0.071	0.228	0.274	0.272	0.884	0.300	0.003

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	91	114	91	173	94	921	121	112
N.S.	1	1.12	1.41	1.12	2.14	1.16	11.37	1.49	1.38
time (sec)	N/A	0.355	0.425	0.291	0.265	0.295	1.933	0.284	6.321

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	121	133	117	253	130	1695	147	187
N.S.	1	1.14	1.25	1.10	2.39	1.23	15.99	1.39	1.76
time (sec)	N/A	0.485	0.254	0.325	0.278	0.271	4.403	0.293	0.003

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	42	63	36	49	46	42	36	19
N.S.	1	0.67	1.00	0.57	0.78	0.73	0.67	0.57	0.30
time (sec)	N/A	0.207	0.031	0.148	0.182	0.274	0.330	0.305	6.033

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	70	126	68	115	88	466	81	64
N.S.	1	0.80	1.43	0.77	1.31	1.00	5.30	0.92	0.73
time (sec)	N/A	0.282	0.171	0.254	0.184	0.279	0.716	0.318	5.720

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	100	180	100	195	133	1227	107	115
N.S.	1	0.88	1.59	0.88	1.73	1.18	10.86	0.95	1.02
time (sec)	N/A	0.394	0.356	0.277	0.183	0.263	1.390	0.326	7.216

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	130	235	132	275	181	2356	133	168
N.S.	1	0.94	1.70	0.96	1.99	1.31	17.07	0.96	1.22
time (sec)	N/A	0.534	0.640	0.346	0.193	0.308	3.129	0.323	8.675

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	44	65	36	49	46	42	36	19
N.S.	1	0.68	1.00	0.55	0.75	0.71	0.65	0.55	0.29
time (sec)	N/A	0.204	0.033	0.148	0.191	0.258	0.339	0.311	5.726

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	72	130	68	115	88	462	81	64
N.S.	1	0.80	1.44	0.76	1.28	0.98	5.13	0.90	0.71
time (sec)	N/A	0.283	0.214	0.256	0.188	0.281	0.746	0.301	6.107

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	102	184	100	195	133	1224	107	116
N.S.	1	0.89	1.60	0.87	1.70	1.16	10.64	0.93	1.01
time (sec)	N/A	0.405	0.522	0.273	0.193	0.280	1.432	0.308	7.675

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	132	241	132	275	181	2353	133	168
N.S.	1	0.94	1.72	0.94	1.96	1.29	16.81	0.95	1.20
time (sec)	N/A	0.531	0.713	0.343	0.193	0.297	3.153	0.325	9.100

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	44	65	36	49	46	42	36	19
N.S.	1	0.68	1.00	0.55	0.75	0.71	0.65	0.55	0.29
time (sec)	N/A	0.209	0.027	0.116	0.189	0.281	0.348	0.301	5.757

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	72	130	68	115	88	462	81	64
N.S.	1	0.80	1.44	0.76	1.28	0.98	5.13	0.90	0.71
time (sec)	N/A	0.305	0.084	0.148	0.177	0.301	0.724	0.310	0.003

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	102	183	100	195	133	1224	107	115
N.S.	1	0.89	1.59	0.87	1.70	1.16	10.64	0.93	1.00
time (sec)	N/A	0.388	0.374	0.266	0.193	0.270	1.412	0.320	7.333

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	132	241	132	275	181	2353	133	168
N.S.	1	0.94	1.72	0.94	1.96	1.29	16.81	0.95	1.20
time (sec)	N/A	0.515	0.009	0.227	0.206	0.287	3.118	0.324	0.003

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	42	63	36	49	46	44	36	19
N.S.	1	0.67	1.00	0.57	0.78	0.73	0.70	0.57	0.30
time (sec)	N/A	0.199	0.028	0.121	0.213	0.269	0.330	0.303	5.823

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	70	126	68	115	88	468	81	64
N.S.	1	0.80	1.43	0.77	1.31	1.00	5.32	0.92	0.73
time (sec)	N/A	0.280	0.007	0.194	0.186	0.290	0.723	0.303	0.003

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	100	179	100	195	133	1229	107	116
N.S.	1	0.88	1.58	0.88	1.73	1.18	10.88	0.95	1.03
time (sec)	N/A	0.393	0.462	0.267	0.195	0.267	1.417	0.309	7.303

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	130	235	132	275	181	2358	133	168
N.S.	1	0.94	1.70	0.96	1.99	1.31	17.09	0.96	1.22
time (sec)	N/A	0.508	0.518	0.230	0.202	0.300	3.127	0.304	0.003

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	267	220	1040	0	491	0	0	0
N.S.	1	1.04	0.86	4.06	0.00	1.92	0.00	0.00	0.00
time (sec)	N/A	1.332	0.759	1.193	0.000	0.127	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	213	185	890	0	447	0	0	0
N.S.	1	1.03	0.89	4.30	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.985	0.720	0.802	0.000	0.125	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	168	142	656	0	404	0	0	0
N.S.	1	1.01	0.85	3.93	0.00	2.42	0.00	0.00	0.00
time (sec)	N/A	0.733	0.621	0.585	0.000	0.101	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	239	0	363	0	0	55
N.S.	1	1.00	0.98	3.85	0.00	5.85	0.00	0.00	0.89
time (sec)	N/A	0.271	2.374	0.911	0.000	0.098	0.000	0.000	5.812

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	126	0	148	0	0	55
N.S.	1	1.00	0.98	2.03	0.00	2.39	0.00	0.00	0.89
time (sec)	N/A	0.274	0.084	0.281	0.000	0.093	0.000	0.000	6.288

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	87	443	0	486	0	0	0
N.S.	1	1.00	0.78	3.99	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.402	0.240	0.373	0.000	0.107	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	241	166	497	0	731	0	0	0
N.S.	1	1.04	0.72	2.15	0.00	3.16	0.00	0.00	0.00
time (sec)	N/A	1.045	0.709	0.617	0.000	0.116	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	318	198	584	0	1051	0	0	0
N.S.	1	1.09	0.68	2.00	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	1.411	0.966	0.872	0.000	0.161	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	244	0	0	0	0	0	0
N.S.	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	1.180	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	118	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	118	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	118	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	116	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	262	0	0	0	0	0	0
N.S.	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	1.354	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	120	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.218	0.000	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	83	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.120	0.000	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	99	0	0	0	0	0	0
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	96	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	83	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	84	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	95	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	100	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.102	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [50] had the largest ratio of [1.28570999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	8	1.07	14	0.571
2	A	6	6	1.04	14	0.429
3	A	4	4	1.00	14	0.286
4	A	2	2	1.00	14	0.143
5	A	4	3	1.00	14	0.214
6	A	6	5	1.00	14	0.357
7	A	8	7	1.05	14	0.500
8	A	4	4	1.00	14	0.286
9	A	4	4	1.00	14	0.286
10	A	4	4	1.00	14	0.286
11	A	4	4	1.00	14	0.286
12	A	4	4	1.00	14	0.286
13	A	4	4	1.00	14	0.286
14	A	4	4	1.00	12	0.333
15	A	4	4	1.00	13	0.308
16	A	2	2	1.00	12	0.167
17	A	2	2	1.00	12	0.167
18	A	2	2	1.00	12	0.167
19	A	5	5	1.09	12	0.417
20	A	8	8	1.12	12	0.667
21	A	11	11	1.14	12	0.917
22	A	2	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	5	5	1.09	12	0.417
24	A	8	8	1.12	12	0.667
25	A	11	11	1.14	12	0.917
26	A	2	2	1.00	12	0.167
27	A	5	5	1.09	12	0.417
28	A	7	7	1.12	12	0.583
29	A	11	11	1.14	12	0.917
30	A	2	2	1.00	12	0.167
31	A	5	5	1.09	12	0.417
32	A	7	7	1.12	12	0.583
33	A	11	11	1.14	12	0.917
34	A	5	4	0.67	12	0.333
35	A	8	7	0.80	12	0.583
36	A	11	10	0.88	12	0.833
37	A	14	13	0.94	12	1.083
38	A	5	4	0.68	12	0.333
39	A	8	7	0.80	12	0.583
40	A	11	10	0.89	12	0.833
41	A	14	13	0.94	12	1.083
42	A	5	4	0.68	12	0.333
43	A	8	7	0.80	12	0.583
44	A	10	9	0.89	12	0.750
45	A	14	13	0.94	12	1.083
46	A	5	4	0.67	12	0.333
47	A	8	7	0.80	12	0.583
48	A	10	9	0.88	12	0.750
49	A	14	13	0.94	12	1.083
50	A	18	18	1.04	14	1.286
51	A	15	15	1.03	14	1.071
52	A	12	12	1.01	14	0.857
53	A	4	4	1.00	14	0.286
54	A	4	4	1.00	14	0.286
55	A	7	7	1.00	14	0.500
56	A	15	15	1.04	14	1.071

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	18	18	1.09	14	1.286
58	A	5	4	1.00	14	0.286
59	A	5	4	1.00	14	0.286
60	A	5	4	1.00	14	0.286
61	A	5	4	1.00	14	0.286
62	A	5	4	1.00	14	0.286
63	A	5	4	1.00	14	0.286
64	A	5	4	1.00	12	0.333
65	A	4	3	1.00	12	0.250
66	A	4	3	1.00	12	0.250
67	A	4	3	1.00	12	0.250
68	A	4	3	1.00	12	0.250
69	A	4	3	1.00	12	0.250
70	A	4	3	1.00	12	0.250
71	A	5	4	1.00	12	0.333
72	A	6	5	1.00	12	0.417

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+a\sin(c+dx))^{7/2}\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 119

$$\int (a+a\sin(c+dx))^{7/2}\,dx = -\frac{256a^4\cos(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{64a^3\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d} - \frac{24a^2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35d} - \frac{2a\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7d}$$

output
```
-24/35*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/7*a*cos(d*x+c)*(a+a*sin(d
*x+c))^(5/2)/d-256/35*a^4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-64/35*a^3*co
s(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

3.1.2 Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.29

$$\int (a+a\sin(c+dx))^{7/2}\,dx = -\frac{a^3(1+\sin(c+dx))^3\sqrt{a(1+\sin(c+dx))}\left(1225\cos\left(\frac{1}{2}(c+dx)\right)+245\cos\left(\frac{3}{2}(c+dx)\right)-49\cos\left(\frac{5}{2}(c+dx)\right.\right.}{140d\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(7/2),x]
```

output
```
-1/140*(a^3*(1 + Sin[c + d*x])^3*Sqrt[a*(1 + Sin[c + d*x])]*(1225*Cos[(c +
 d*x)/2] + 245*Cos[(3*(c + d*x))/2] - 49*Cos[(5*(c + d*x))/2] - 5*Cos[(7*(
c + d*x))/2] - 1225*Sin[(c + d*x)/2] + 245*Sin[(3*(c + d*x))/2] + 49*Sin[(
5*(c + d*x))/2] - 5*Sin[(7*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^7)
```

3.1.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3126, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^{7/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{12}{7}a\int (\sin(c+dx)a+a)^{5/2}dx-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\int (\sin(c+dx)a+a)^{5/2}dx-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int (\sin(c+dx)a+a)^{3/2}dx-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int (\sin(c+dx)a+a)^{3/2}dx-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \; 3126$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(c+dx)a+a}dx-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(c+dx)a+a}dx-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \; 3125$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7d}$$

input
```
Int[(a + a*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*a*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*d) + (12*a*((-2*a*Cos[c
+ d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*d) + (8*a*((-8*a^2*Cos[c + d*x])/(3*
d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/
(3*d)))/5))/7
```

3.1.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.1.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2(1+\sin(dx+c))a^4(\sin(dx+c)-1)(5(\sin^3(dx+c))+27(\sin^2(dx+c))+71\sin(dx+c)+177)}{35\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	75

input
```
int((a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)*(5*sin(d*x+c)^3+27*sin(d*x+c)^2+71*
sin(d*x+c)+177)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int(a+a\sin(c+dx))^{7/2}\,dx=\frac{2\left(5\,a^3\cos\left(dx+c\right)^4+27\,a^3\cos\left(dx+c\right)^3-54\,a^3\cos\left(dx+c\right)^2-204\,a^3\cos\left(dx+c\right)-128\,a^3 \cdots\right.}{35\left(d\cos\left(dx \cdots\right.\right.}$$

input
```
integrate((a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^3*cos(d*x + c)^4 + 27*a^3*cos(d*x + c)^3 - 54*a^3*cos(d*x + c)^2
 - 204*a^3*cos(d*x + c) - 128*a^3 + (5*a^3*cos(d*x + c)^3 - 22*a^3*cos(d*x
 + c)^2 - 76*a^3*cos(d*x + c) + 128*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c)
 + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

3.1.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.1.7 Maxima [F]

$$\int (a+a\sin(c+dx))^{7/2}\,dx=\int (a\sin(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(7/2), x)
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int (a+a\sin(c+dx))^{7/2}\,dx=\frac{\sqrt{2}\left(1225\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+245\,a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}c\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
1/140*sqrt(2)*(1225*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c) + 245*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi
 + 3/2*d*x + 3/2*c) + 49*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*
pi + 5/2*d*x + 5/2*c) + 5*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4
*pi + 7/2*d*x + 7/2*c))*sqrt(a)/d
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^{7/2}\,dx = \int (a + a\ \sin(c + d\,x))^{7/2}\,dx$$

input
```
int((a + a*sin(c + d*x))^(7/2),x)
```

output
```
int((a + a*sin(c + d*x))^(7/2), x)
```

3.2 $\int (a+a\sin(c+dx))^{5/2}\,dx$

3.2.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (a+a\sin(c+dx))^{5/2}\,dx=-\frac{64a^3\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}-\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}$$

output
```
-2/5*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-64/15*a^3*cos(d*x+c)/d/(a+a*sin
(d*x+c))^(1/2)-16/15*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

3.2.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

$$\int (a+a\sin(c+dx))^{5/2}\,dx=$$

$$-\frac{(a(1+\sin(c+dx)))^{5/2}\left(150\cos\left(\frac{1}{2}(c+dx)\right)+25\cos\left(\frac{3}{2}(c+dx)\right)-3\cos\left(\frac{5}{2}(c+dx)\right)-150\sin\left(\frac{1}{2}(c+dx\right.\right.}{30d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(5/2),x]
```

output
```
-1/30*((a*(1 + Sin[c + d*x]))^(5/2)*(150*Cos[(c + d*x)/2] + 25*Cos[(3*(c +
 d*x))/2] - 3*Cos[(5*(c + d*x))/2] - 150*Sin[(c + d*x)/2] + 25*Sin[(3*(c +
 d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^5)
```

3.2.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (\sin(c+dx)a+a)^{3/2}dx-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int (\sin(c+dx)a+a)^{3/2}dx-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(c+dx)a+a}dx-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(c+dx)a+a}dx-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{5}a\left(-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}\right)-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[(a + a*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*a*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*d) + (8*a*((-8*a^2*Cos[c
 + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sqrt[a + a*Sin
[c + d*x]])/(3*d)))/5
```

3.2.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.2.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{2(1+\sin(dx+c))a^3(\sin(dx+c)-1)(3(\sin^2(dx+c))+14\sin(dx+c)+43)}{15\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	65

input
```
int((a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(3*sin(d*x+c)^2+14*sin(d*x+c)+43)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.29

$$\int (a + a\sin(c + dx))^{5/2}\,dx = \frac{2\left(3\,a^2\cos\left(dx+c\right)^3 - 11\,a^2\cos\left(dx+c\right)^2 - 46\,a^2\cos\left(dx+c\right) - 32\,a^2 - \left(3\,a^2\cos\left(dx+c\right)^2 + \right.\right.}{15\left(d\cos\left(dx+c\right) + d\sin\left(dx+c\right) + \right.}$$

input
```
integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*cos(d*x + c)^3 - 11*a^2*cos(d*x + c)^2 - 46*a^2*cos(d*x + c) -
 32*a^2 - (3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) - 32*a^2)*sin(d*x +
c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

3.2.6 Sympy [F]

$$\int (a + a\sin(c + dx))^{5/2}\,dx = \int \left(a\sin\left(c+dx\right) + a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(5/2), x)
```

3.2.7 Maxima [F]

$$\int (a + a\sin(c + dx))^{5/2}\,dx = \int \left(a\sin\left(dx+c\right) + a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(5/2), x)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int (a+a\sin(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(150\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx\right.\right.\right.}{3}$$

input
```
integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/30*sqrt(2)*(150*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c) + 25*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi +
3/2*d*x + 3/2*c) + 3*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi +
 5/2*d*x + 5/2*c))*sqrt(a)/d
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^{5/2}\,dx = \int (a+a\,\sin(c+d\,x))^{5/2}\,dx$$

input
```
int((a + a*sin(c + d*x))^(5/2),x)
```

output
```
int((a + a*sin(c + d*x))^(5/2), x)
```

3.3 $\int (a+a\sin(c+dx))^{3/2}\,dx$

3.3.1 Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (a+a\sin(c+dx))^{3/2}\,dx=-\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}$$

output
```
-8/3*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*a*cos(d*x+c)*(a+a*sin(d*x
+c))^(1/2)/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.51

$$\int (a+a\sin(c+dx))^{3/2}\,dx = -\frac{(a(1+\sin(c+dx)))^{3/2}\left(9\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)-9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(3/2),x]
```

output
```
-1/3*((a*(1 + Sin[c + d*x]))^(3/2)*(9*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))
/2] - 9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^3)
```

3.3.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{4}{3}a\int \sqrt{\sin(c+dx)a+a}dx-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sqrt{\sin(c+dx)a+a}dx-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3125}$$

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

input
```
Int[(a + a*Sin[c + d*x])^(3/2),x]
```

output
```
(-8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(3*d)
```

3.3.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)(\sin(dx+c)+5)}{3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	53

input
```
int((a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(sin(d*x+c)+5)/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int (a+a\sin(c+dx))^{3/2}\,dx =$$
$$-\frac{2\left(a\cos\left(dx+c\right)^2+5\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)-4\,a\right)\sin\left(dx+c\right)+4\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c) + (a*cos(d*x + c) - 4*a)*sin(d*x
 + c) + 4*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d
)
```

3.3.6 Sympy [F]

$$\int (a+a\sin(c+dx))^{3/2}\,dx = \int \left(a\sin\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(3/2), x)
```

3.3.7 Maxima [F]

$$\int (a+a\sin(c+dx))^{3/2}\,dx = \int \left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(3/2), x)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int (a+a\sin(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(9\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\cdots}{3\,d}$$

input
```
integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*(9*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c) + a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*x +
3/2*c))*sqrt(a)/d
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^{3/2}\,dx=\int (a+a\,\sin(c+d\,x))^{3/2}\,dx$$

input
```
int((a + a*sin(c + d*x))^(3/2),x)
```

output
```
int((a + a*sin(c + d*x))^(3/2), x)
```

3.4 $\int \sqrt{a+a\sin(c+dx)}\,dx$

3.4.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a+a\sin(c+dx)}\,dx=-\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

output
```
-2*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

3.4.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \sqrt{a+a\sin(c+dx)}\,dx=\frac{2\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(2*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

3.4.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sin(c + dx) + a}\, dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a \sin(c + dx) + a} dx$$

$$\Big\downarrow \; 3125$$

$$-\frac{2a \cos(c + dx)}{d\sqrt{a \sin(c + dx) + a}}$$

input
```
Int[Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
(-2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

3.4.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)a}{\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	43
risch	$-\frac{i\sqrt{2}\,\sqrt{-a(-2-2\sin(dx+c))}\,(-i+e^{i(dx+c)})\,(e^{i(dx+c)}+i)}{(e^{2i(dx+c)}+2ie^{i(dx+c)}-1)\,d}$	74

input
```
int((a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1+sin(d*x+c))*(sin(d*x+c)-1)*a/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\,\sqrt{a\sin{(dx+c)}+a}(\cos{(dx+c)}-\sin{(dx+c)}+1)}{d\cos{(dx+c)}+d\sin{(dx+c)}+d}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/(d*cos(d*x +
 c) + d*sin(d*x + c) + d)
```

3.4.6 Sympy [F]

$$\int \sqrt{a+a\sin(c+dx)}\,dx = \int \sqrt{a\sin{(c+dx)}+a}\,dx$$

input
```
integrate((a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*sin(c + d*x) + a), x)
```

3.4.7 Maxima [F]

$$\int \sqrt{a+a\sin(c+dx)}\,dx = \int \sqrt{a\sin(dx+c)+a}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(d*x + c) + a), x)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \sqrt{a+a\sin(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{d}$$

input
```
integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*
x + 1/2*c)/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos(c+d\,x)\,\sqrt{a\,(\sin(c+d\,x)+1)}}{d\,(\sin(c+d\,x)+1)}$$

input
```
int((a + a*sin(c + d*x))^(1/2),x)
```

output
```
-(2*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(d*(sin(c + d*x) + 1))
```

3.5 $\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$

3.5.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx=-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
-arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/
a^(1/2)
```

3.5.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$=\frac{(2+2i)(-1)^{3/4}\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\sin(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4
])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

3.5.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3128}$$

$$-\frac{2\int \frac{1}{2a-\frac{a^2\cos^2(c+dx)}{\sin(c+dx)a+a}}d\frac{a\cos(c+dx)}{\sqrt{\sin(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Sin[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(Sqrt[a]*d))
```

3.5.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

method	result	si
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	75
risch	$\frac{2i\left(\mathrm{e}^{i(dx+c)}+i\right)\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{d\sqrt{-a\left(i\mathrm{e}^{2i(dx+c)}-i-2\,\mathrm{e}^{i(dx+c)}\right)\mathrm{e}^{-i(dx+c)}}}-\frac{2i\left(\mathrm{e}^{i(dx+c)}+i\right)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{-ia\,\mathrm{e}^{i(dx+c)}}}{\sqrt{a}}\right)a\sqrt{-ia\,\mathrm{e}^{i(dx+c)}}\right)\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{d\,a^{\frac{3}{2}}\sqrt{-a\left(i\mathrm{e}^{2i(dx+c)}-i-2\,\mathrm{e}^{i(dx+c)}\right)\mathrm{e}^{-i(dx+c)}}}$	19

input
```
int(1/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2*(-a*
(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.55

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{2\sqrt{a}d}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\,\mathrm{arctan}\right.$$

input
```
integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - c
os(d*x + c) - 2))/(sqrt(a)*d), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*si
n(d*x + c) + a)*sqrt(-1/a)/cos(d*x + c))/d]
```

3.5.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(c+dx\right)+a}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*sin(c + d*x) + a), x)
```

3.5.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*sin(d*x + c) + a), x)
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(38) = 76$.

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{4\,d}$$

input
```
integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*log(abs(1/sin(-1/4*pi + 1/2*d*x + 1/2*c) + sin(-1/4*pi + 1/2*
d*x + 1/2*c) + 2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - sqrt(2)
*log(abs(1/sin(-1/4*pi + 1/2*d*x + 1/2*c) + sin(-1/4*pi + 1/2*d*x + 1/2*c)
 - 2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx=-\frac{\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\sin(c+d\,x))}{a}}}{d\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}$$

input
```
int(1/(a + a*sin(c + d*x))^(1/2),x)
```

output
```
-(ellipticF(pi/4 - c/2 - (d*x)/2, 1)*((2*(a + a*sin(c + d*x)))/a)^(1/2))/(
d*(a + a*sin(c + d*x))^(1/2))
```

3.6 $\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx$

3.6.1 Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

output
```
-1/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-1/4*arctanh(1/2*cos(d*x+c)*a^(1/2
)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

3.6.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)+(\cdots}{2d(a(1+\mathrm{s}\cdots}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(-3/2),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/
2] + (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])]*(1 + Sin[c + d*x])))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

3.6.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(c+dx)+a)^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\sin(c+dx)+a)^{3/2}}dx$$
$$\Big\downarrow \text{3129}$$
$$\frac{\int \frac{1}{\sqrt{\sin(c+dx)a+a}}dx}{4a}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{\sqrt{\sin(c+dx)a+a}}dx}{4a}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$
$$\Big\downarrow \text{3128}$$
$$-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(c+dx)}{\sin(c+dx)a+a}}d\frac{a\cos(c+dx)}{\sqrt{\sin(c+dx)a+a}}}{2ad}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

input
```
Int[(a + a*Sin[c + d*x])^(-3/2),x]
```

output
```
-1/2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(S
qrt[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

3.6.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.6.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.62

method	result
default	$-\frac{\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\sin(dx+c)+2\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\right)\sqrt{-a(\sin(dx+c)-}}{4a^{\frac{7}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

input
```
int(1/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a^2*sin(d*x+c)+2*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a-a*s
in(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c
)/(a+a*sin(d*x+c))^(1/2)/d
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.27

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log}{8\left(a^2d\cos\right.}$$

input
```
integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x +
 c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d
*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*si
n(d*x + c) - cos(d*x + c) - 2)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)
 - sin(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d
 - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

3.6.6 Sympy [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\sin\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(-3/2), x)
```

3.6.7 Maxima [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(-3/2), x)
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(62) = 124$.

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{2\,\sin\left(-\frac{1}{4}\right.}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right.}\right.}{8\,\sqrt{a}d}$$

input
```
integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))) - log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) - 2*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((sin(-1/4*
pi + 1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/(sqrt
(a)*d)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + a*sin(c + d*x))^(3/2),x)
```

output
```
int(1/(a + a*sin(c + d*x))^(3/2), x)
```

3.7 $\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx$

3.7.1 Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{3\text{arctanh}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

output
```
-1/4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)-3/16*cos(d*x+c)/a/d/(a+a*sin(d*x+
c))^(3/2)-3/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/
2))/a^(5/2)/d*2^(1/2)
```

3.7.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)-4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(-5/2),x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 6*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 - 3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (3 + 3*I)*(-1)
^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

3.7.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3129, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a\sin(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{3\int \frac{1}{(\sin(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{1}{(\sin(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(c+dx)a+a}}dx}{4a}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(c+dx)a+a}}dx}{4a}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{3\left(-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(c+dx)}{\sin(c+dx)a+a}}d\frac{a\cos(c+dx)}{\sqrt{\sin(c+dx)a+a}}}{2ad}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{3\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

input
```
Int[(a + a*Sin[c + d*x])^(-5/2),x]
```

output
```
-1/4*Cos[c + d*x]/(d*(a + a*Sin[c + d*x])^(5/2)) + (3*(-1/2*ArcTanh[(Sqrt[
a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(Sqrt[2]*a^(3/2)*d) -
 Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))))/(8*a)
```

3.7.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\left(\cos^{2}(dx+c)\right)+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\sin(dx+c)+6\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}\,\mathrm{s}}{32a^{\frac{9}{2}}\left(1+\sin(dx+c)\right)\cos(dx+c)\sqrt{a+a\sin(}}$

input
```
int(1/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^(9/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^2*cos(d*x+c)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2))*a^2*sin(d*x+c)+6*(a-a*sin(d*x+c))^(1/2)*a^(3/2)*sin(d*x+c)+6*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+14*(a-a*sin(d*
x+c))^(1/2)*a^(3/2))*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(88) = 176$.

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.99

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\mathrm{s}\right.}{}$$

input
```
integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(
d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2
*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))
 + 4*(3*cos(d*x + c)^2 + (3*cos(d*x + c) - 4)*sin(d*x + c) + 7*cos(d*x + c
) + 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
 c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*c
os(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

3.7.6 Sympy [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx=\int \frac{1}{(a\sin{(c+dx)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(-5/2), x)
```

3.7.7 Maxima [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx=\int \frac{1}{(a\sin{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(-5/2), x)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx=\frac{\sqrt{2}\left(\frac{3\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{3\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\left(3\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx\right.\right.}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}\right.}{64\,\sqrt{a}d}$$

input
```
integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/64*sqrt(2)*(3*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))) - 3*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*(3*sin(-1/4*pi + 1/2*d*x + 1/2*c)
^3 - 5*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2
- 1)^2*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/(sqrt(a)*d)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a + a*sin(c + d*x))^(5/2),x)
```

output
```
int(1/(a + a*sin(c + d*x))^(5/2), x)
```

3.8 $\int (a+a\sin(c+dx))^{4/3}\,dx$

3.8.1 Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (a+a\sin(c+dx))^{4/3}\,dx = -\frac{2\,2^{5/6}a\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}$$

output
```
-2*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(a
+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

3.8.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 223 vs. 2(67) = 134.

Time = 1.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.33

$$\int (a+a\sin(c+dx))^{4/3}\,dx = \frac{(a(1+\sin(c+dx)))^{4/3}\left(20\sqrt[3]{2}\cos\left(\frac{1}{4}(2c+\pi+2dx)\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)\right.}{8d\sqrt{\cos^2\left(\frac{1}{4}(2c+\pi+2d\right.}}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(4/3),x]
```

output
```
((a*(1 + Sin[c + d*x]))^(4/3)*(20*2^(1/3)*Cos[(2*c + Pi + 2*d*x)/4]*Hyperg
eometricPFQ[{-1/2, -1/6}, {5/6}, Sin[(2*c + Pi + 2*d*x)/4]^2] - Sqrt[2 - 2
*Sin[c + d*x]]*(10*2^(1/3)*Cos[(2*c + Pi + 2*d*x)/4] + 3*Cos[c + d*x]*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^(2/3)*Sin[(2*c + Pi + 2*d*x)/4]^(1/3))))
/(8*d*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^(8/3)*Sin[(2*c + Pi + 2*d*x)/4]^(1/3))
```

3.8.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^{4/3}dx$$

$$\Big\downarrow \text{3131}$$

$$\frac{a\sqrt[3]{a\sin(c+dx)+a}\int (\sin(c+dx)+1)^{4/3}dx}{\sqrt[3]{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt[3]{a\sin(c+dx)+a}\int (\sin(c+dx)+1)^{4/3}dx}{\sqrt[3]{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{3130}$$

$$-\frac{2\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

input
```
Int[(a + a*Sin[c + d*x])^(4/3),x]
```

output
```
(-2*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6))
```

3.8.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.8.4 Maple [F]

$$\int (a + a\sin(dx + c))^{\frac{4}{3}} dx$$

input
```
int((a+a*sin(d*x+c))^(4/3),x)
```

output
```
int((a+a*sin(d*x+c))^(4/3),x)
```

3.8.5 Fricas [F]

$$\int (a + a\sin(c + dx))^{4/3} dx = \int (a\sin(dx + c) + a)^{\frac{4}{3}} dx$$

input
```
integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((a*sin(d*x + c) + a)^(4/3), x)
```

3.8.6 Sympy [F]

$$\int (a+a\sin(c+dx))^{4/3}\,dx = \int (a\sin(c+dx)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))**(4/3),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(4/3), x)
```

3.8.7 Maxima [F]

$$\int (a+a\sin(c+dx))^{4/3}\,dx = \int (a\sin(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(4/3), x)
```

3.8.8 Giac [F]

$$\int (a+a\sin(c+dx))^{4/3}\,dx = \int (a\sin(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^(4/3), x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^{4/3}\, dx = \int (a + a\ \sin(c + d\,x))^{4/3}\, dx$$

input
```
int((a + a*sin(c + d*x))^(4/3),x)
```

output
```
int((a + a*sin(c + d*x))^(4/3), x)
```

3.9 $\int (a + a\sin(c + dx))^{2/3}\, dx$

3.9.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a + a\sin(c + dx))^{2/3}\, dx = -\frac{2\sqrt[6]{2}\cos(c + dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \sin(c + dx))\right)(a + a\sin(c + dx))^{2/3}}{d(1 + \sin(c + dx))^{7/6}}$$

output
```
-2*2^(1/6)*cos(d*x+c)*hypergeom([-1/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+a
*sin(d*x+c))^(2/3)/d/(1+sin(d*x+c))^(7/6)
```

3.9.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\int (a + a\sin(c + dx))^{2/3}\, dx = -\frac{3\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\left(-2\,\text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \sin^2\left(\frac{1}{4}(2c + \pi + 2dx)\right)\right) + \sqrt{2 - 2}\right.}{2d\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\sqrt{2 - 2\sin(c + dx)}}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(2/3),x]
```

output
```
(-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(-2*Hypergeometric2F1[1/6, 1/2,
7/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt[2 - 2*Sin[c + d*x]])*(a*(1 + Sin[
c + d*x]))^(2/3))/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[2 - 2*Si
n[c + d*x]])
```

3.9.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^{2/3}dx$$

$$\Big\downarrow \text{3131}$$

$$\frac{(a\sin(c+dx)+a)^{2/3}\int (\sin(c+dx)+1)^{2/3}dx}{(\sin(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sin(c+dx)+a)^{2/3}\int (\sin(c+dx)+1)^{2/3}dx}{(\sin(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3130}$$

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

input
```
Int[(a + a*Sin[c + d*x])^(2/3),x]
```

output
```
(-2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*
x])/2]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

3.9.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.9.4 Maple [F]

$$\int (a+a\sin (dx+c))^{\frac{2}{3}}\, dx$$

input
```
int((a+a*sin(d*x+c))^(2/3),x)
```

output
```
int((a+a*sin(d*x+c))^(2/3),x)
```

3.9.5 Fricas [F]

$$\int (a+a\sin(c+dx))^{2/3}\, dx = \int (a\sin (dx+c)+a)^{\frac{2}{3}}\, dx$$

input
```
integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sin(d*x + c) + a)^(2/3), x)
```

3.9.6 Sympy [F]

$$\int (a+a\sin(c+dx))^{2/3}\,dx=\int (a\sin{(c+dx)}+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))**(2/3),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(2/3), x)
```

3.9.7 Maxima [F]

$$\int (a+a\sin(c+dx))^{2/3}\,dx=\int (a\sin{(dx+c)}+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(2/3), x)
```

3.9.8 Giac [F]

$$\int (a+a\sin(c+dx))^{2/3}\,dx=\int (a\sin{(dx+c)}+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^(2/3), x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^{2/3}\,dx = \int (a+a\,\sin(c+d\,x))^{2/3}\,dx$$

input
```
int((a + a*sin(c + d*x))^(2/3),x)
```

output
```
int((a + a*sin(c + d*x))^(2/3), x)
```

3.10 $\int \sqrt[3]{a+a\sin(c+dx)}\,dx$

3.10.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \sqrt[3]{a+a\sin(c+dx)}\,dx = -\frac{2^{5/6}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}$$

output
```
-2^(5/6)*cos(d*x+c)*hypergeom([1/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+a*si
n(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

3.10.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(66) = 132.

Time = 0.40 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.56

$$\int \sqrt[3]{a+a\sin(c+dx)}\,dx = \frac{\sqrt[3]{2}\left(2\cos\left(\frac{1}{4}(2c+\pi+2dx)\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)+\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{d\sqrt{\cos^2\left(\frac{1}{4}(2c+\pi+2dx)\right)}\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{2/3}\sqrt[3]{\sin\left(\frac{1}{4}(\right.}}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(1/3),x]
```

output
```
(2^(1/3)*(2*Cos[(2*c + Pi + 2*d*x)/4]*HypergeometricPFQ[{-1/2, -1/6}, {5/6
}, Sin[(2*c + Pi + 2*d*x)/4]^2] + (-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*S
qrt[1 - Sin[c + d*x]])*(a*(1 + Sin[c + d*x]))^(1/3))/(d*Sqrt[Cos[(2*c + Pi
 + 2*d*x)/4]^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(2/3)*Sin[(2*c + Pi
+ 2*d*x)/4]^(1/3))
```

3.10.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3131}$$

$$\frac{\sqrt[3]{a\sin(c+dx)+a}\int \sqrt[3]{\sin(c+dx)+1}dx}{\sqrt[3]{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{a\sin(c+dx)+a}\int \sqrt[3]{\sin(c+dx)+1}dx}{\sqrt[3]{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{3130}$$

$$-\frac{2^{5/6}\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

input
```
Int[(a + a*Sin[c + d*x])^(1/3),x]
```

output
```
-((2^(5/6)*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Sin[c + d*x]
)/2]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)))
```

3.10.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.10.4 Maple [F]

$$\int (a + a\sin(dx + c))^{\frac{1}{3}}\,dx$$

input
```
int((a+a*sin(d*x+c))^(1/3),x)
```

output
```
int((a+a*sin(d*x+c))^(1/3),x)
```

3.10.5 Fricas [F]

$$\int \sqrt[3]{a + a\sin(c + dx)}\,dx = \int (a\sin(dx + c) + a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((a*sin(d*x + c) + a)^(1/3), x)
```

3.10.6 Sympy [F]

$$\int \sqrt[3]{a+a\sin(c+dx)}\,dx = \int \sqrt[3]{a\sin\left(c+dx\right)+a}\,dx$$

input
```
integrate((a+a*sin(d*x+c))**(1/3),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(1/3), x)
```

3.10.7 Maxima [F]

$$\int \sqrt[3]{a+a\sin(c+dx)}\,dx = \int \left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(1/3), x)
```

3.10.8 Giac [F]

$$\int \sqrt[3]{a+a\sin(c+dx)}\,dx = \int \left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^(1/3), x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\sin(c+dx)}\,dx = \int (a+a\,\sin(c+d\,x))^{1/3}\,dx$$

input
```
int((a + a*sin(c + d*x))^(1/3),x)
```

output
```
int((a + a*sin(c + d*x))^(1/3), x)
```

3.11 $\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

3.11.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = -\frac{\sqrt[6]{2}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

output
```
-2^(1/6)*cos(d*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*sin(d*x+c))/d/(1+si
n(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

3.11.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = \frac{3\sqrt{2}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2\left(\frac{1}{4}(2c+\pi+2dx)\right)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt[3]{a(1+\sin(c+dx))}}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(-1/3),x]
```

output
```
(3*Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2
*d*x)/4]^2])/(d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

3.11.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt[3]{a\sin(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3131}$$

$$\frac{\sqrt[3]{\sin(c+dx)+1}\int \frac{1}{\sqrt[3]{\sin(c+dx)+1}}dx}{\sqrt[3]{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[3]{\sin(c+dx)+1}\int \frac{1}{\sqrt[3]{\sin(c+dx)+1}}dx}{\sqrt[3]{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{ 3130}$$

$$-\frac{\sqrt[6]{2}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

input
```
Int[(a + a*Sin[c + d*x])^(-1/3),x]
```

output
```
-((2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x]
)/2])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

3.11.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.11.4 Maple [F]

$$\int \frac{1}{(a+a\sin(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(1/(a+a*sin(d*x+c))^(1/3),x)
```

output
```
int(1/(a+a*sin(d*x+c))^(1/3),x)
```

3.11.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = \int \frac{1}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((a*sin(d*x + c) + a)^(-1/3), x)
```

3.11.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{a\sin\left(c+dx\right)+a}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))**(1/3),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(-1/3), x)
```

3.11.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx=\int \frac{1}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

3.11.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx=\int \frac{1}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = \int \frac{1}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int(1/(a + a*sin(c + d*x))^(1/3),x)
```

output
```
int(1/(a + a*sin(c + d*x))^(1/3), x)
```

3.12 $\int \frac{1}{(a+a\sin(c+dx))^{2/3}}\,dx$

3.12.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{(a+a\sin(c+dx))^{2/3}}\,dx = -\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{\sqrt[6]{2}d(a+a\sin(c+dx))^{2/3}}$$

output
```
-1/2*cos(d*x+c)*hypergeom([1/2, 7/6],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+
c))^(1/6)*2^(5/6)/d/(a+a*sin(d*x+c))^(2/3)
```

3.12.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(66) = 132.

Time = 0.65 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.26

$$\int \frac{1}{(a+a\sin(c+dx))^{2/3}}\,dx = \frac{2\left(-3\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{(\cos(\frac{1}{2}(c+dx))+\sin(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(-2/3),x]
```

output
```
(2*(-3*Cos[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + ((Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^(4/3)*(-2*Cos[(2*c + Pi + 2*d*x)/4]*Hypergeom
etricPFQ[{-1/2, -1/6}, {5/6}, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt[Cos[(2*c
 + Pi + 2*d*x)/4]^2]*(2*Cos[(2*c + Pi + 2*d*x)/4] + 3*Sin[(2*c + Pi + 2*d*
x)/4])))/(2^(1/6)*Sqrt[1 - Sin[c + d*x]]*Sin[(2*c + Pi + 2*d*x)/4]^(1/3)))
)/(d*(a*(1 + Sin[c + d*x]))^(2/3))
```

3.12.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(c+dx)+a)^{2/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a\sin(c+dx)+a)^{2/3}}dx$$

$$\Big\downarrow \text{ 3131}$$

$$\frac{(\sin(c+dx)+1)^{2/3}\int \frac{1}{(\sin(c+dx)+1)^{2/3}}dx}{(a\sin(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(\sin(c+dx)+1)^{2/3}\int \frac{1}{(\sin(c+dx)+1)^{2/3}}dx}{(a\sin(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{ 3130}$$

$$-\frac{\sqrt[6]{\sin(c+dx)+1}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{\sqrt[6]{2}d(a\sin(c+dx)+a)^{2/3}}$$

input
```
Int[(a + a*Sin[c + d*x])^(-2/3),x]
```

output
```
-((Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(1/6))/(2^(1/6)*d*(a + a*Sin[c + d*x])^(2/3)))
```

3.12.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.12.4 Maple [F]

$$\int \frac{1}{(a+a\sin(dx+c))^{\frac{2}{3}}}dx$$

input
```
int(1/(a+a*sin(d*x+c))^(2/3),x)
```

output
```
int(1/(a+a*sin(d*x+c))^(2/3),x)
```

3.12.5 Fricas [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{2/3}}\,dx=\int \frac{1}{(a\sin(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sin(d*x + c) + a)^(-2/3), x)
```

3.12.6 Sympy [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(a\sin{(c+dx)}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))**(2/3),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(-2/3), x)
```

3.12.7 Maxima [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(a\sin{(dx+c)}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(-2/3), x)
```

3.12.8 Giac [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(a\sin{(dx+c)}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^(-2/3), x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sin(c + dx))^{2/3}}\,dx = \int \frac{1}{(a + a\,\sin(c + d\,x))^{2/3}}\,dx$$

input
```
int(1/(a + a*sin(c + d*x))^(2/3),x)
```

output
```
int(1/(a + a*sin(c + d*x))^(2/3), x)
```

3.13 $\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx$

3.13.1 Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx=-\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

output
```
-1/2*cos(d*x+c)*hypergeom([1/2, 11/6],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/a/
d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

3.13.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx=$$
$$-\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{2-2\sin(c+dx)}-2\,\text{Hyper}\right.}{5d\sqrt{2-2\sin(c+dx)}(a(1+\sin(c+dx)))}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(-4/3),x]
```

output
```
(-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])*(Sqrt[2 - 2*Sin[c + d*x]] - 2*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(
2*c + Pi + 2*d*x)/4]^2]*(1 + Sin[c + d*x])))/(5*d*Sqrt[2 - 2*Sin[c + d*x]]
*(a*(1 + Sin[c + d*x]))^(4/3))
```

3.13.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(c+dx)+a)^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(c+dx)+a)^{4/3}}dx$$

$$\Big\downarrow \text{3131}$$

$$\frac{\sqrt[3]{\sin(c+dx)+1}\int \frac{1}{(\sin(c+dx)+1)^{4/3}}dx}{a\sqrt[3]{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\sin(c+dx)+1}\int \frac{1}{(\sin(c+dx)+1)^{4/3}}dx}{a\sqrt[3]{a\sin(c+dx)+a}}$$

$$\Big\downarrow \text{3130}$$

$$-\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

input
```
Int[(a + a*Sin[c + d*x])^(-4/3),x]
```

output
```
-((Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 3/2, (1 - Sin[c + d*x])/2])/(
2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

3.13.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.13.4 Maple [F]

$$\int \frac{1}{(a + a\sin(dx + c))^{\frac{4}{3}}} dx$$

input
```
int(1/(a+a*sin(d*x+c))^(4/3),x)
```

output
```
int(1/(a+a*sin(d*x+c))^(4/3),x)
```

3.13.5 Fricas [F]

$$\int \frac{1}{(a + a\sin(c + dx))^{4/3}} dx = \int \frac{1}{(a\sin(dx + c) + a)^{\frac{4}{3}}} dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral(-(a*sin(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x +
 c) - 2*a^2), x)
```

3.13.6 Sympy [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(a\sin{(c+dx)}+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))**(4/3),x)
```

output
```
Integral((a*sin(c + d*x) + a)**(-4/3), x)
```

3.13.7 Maxima [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(a\sin{(dx+c)}+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

3.13.8 Giac [F]

$$\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(a\sin{(dx+c)}+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{4/3}}\,dx$$

input
```
int(1/(a + a*sin(c + d*x))^(4/3),x)
```

output
```
int(1/(a + a*sin(c + d*x))^(4/3), x)
```

3.14 $\int (a+a\sin(c+dx))^n\,dx$

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 74

$$\int (a+a\sin(c+dx))^n\,dx = -\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n}{d}$$

output
```
-2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+
sin(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c))^n/d
```

3.14.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int (a+a\sin(c+dx))^n\,dx = \frac{2^n B_{\frac{1}{2}(1+\sin(c+dx))}\left(\frac{1}{2}+n,\frac{1}{2}\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)(1+\sin(c+dx))^{-n}(a(1+\sin(c+dx)))^n}{d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^n,x]
```

output
```
(2^n*Beta[(1 + Sin[c + d*x])/2, 1/2 + n, 1/2]*Sqrt[Cos[c + d*x]^2]*Sec[c +
 d*x]*(a*(1 + Sin[c + d*x]))^n)/(d*(1 + Sin[c + d*x])^n)
```

3.14.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^n dx$$

$$\Big\downarrow \text{3131}$$

$$(\sin(c+dx)+1)^{-n}(a\sin(c+dx)+a)^n\int (\sin(c+dx)+1)^n dx$$

$$\Big\downarrow \text{3042}$$

$$(\sin(c+dx)+1)^{-n}(a\sin(c+dx)+a)^n\int (\sin(c+dx)+1)^n dx$$

$$\Big\downarrow \text{3130}$$

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^n,x]
```

output
```
-((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[
c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/d)
```

3.14.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.14.4 Maple [F]

$$\int (a + a\sin(dx + c))^n \, dx$$

input
```
int((a+a*sin(d*x+c))^n,x)
```

output
```
int((a+a*sin(d*x+c))^n,x)
```

3.14.5 Fricas [F]

$$\int (a + a\sin(c + dx))^n \, dx = \int (a\sin(dx + c) + a)^n \, dx$$

input
```
integrate((a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sin(d*x + c) + a)^n, x)
```

3.14.6 Sympy [F]

$$\int (a + a\sin(c + dx))^n \, dx = \int (a\sin(c + dx) + a)^n \, dx$$

input
```
integrate((a+a*sin(d*x+c))**n,x)
```

output
```
Integral((a*sin(c + d*x) + a)**n, x)
```

3.14.7 Maxima [F]

$$\int (a + a\sin(c + dx))^n \, dx = \int (a\sin(dx + c) + a)^n \, dx$$

input
```
integrate((a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sin(d*x + c) + a)^n, x)
```

3.14.8 Giac [F]

$$\int (a + a\sin(c + dx))^n \, dx = \int (a\sin(dx + c) + a)^n \, dx$$

input
```
integrate((a+a*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + a)^n, x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^n \, dx = \int (a + a\ \sin(c + d\,x))^n \, dx$$

input
```
int((a + a*sin(c + d*x))^n,x)
```

output
```
int((a + a*sin(c + d*x))^n, x)
```

3.15 $\int (a-a\sin(c+dx))^n\,dx$

3.15.1 Optimal result

Integrand size = 13, antiderivative size = 74

$$\int (a-a\sin(c+dx))^n\,dx$$
$$=\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c+dx))^{-\frac{1}{2}-n}(a-a\sin(c}{d}$$

output
```
2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2+1/2*sin(d*x+c))*(1-s
in(d*x+c))^(-1/2-n)*(a-a*sin(d*x+c))^n/d
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int (a-a\sin(c+dx))^n\,dx =$$
$$-\frac{2^n B_{\frac{1}{2}(1-\sin(c+dx))}\left(\frac{1}{2}+n,\frac{1}{2}\right)\sec(c+dx)(1-\sin(c+dx))^{\frac{1}{2}-n}\sqrt{1+\sin(c+dx)}(a-a\sin(c+dx))^n}{d}$$

input
```
Integrate[(a - a*Sin[c + d*x])^n,x]
```

output
```
-((2^n*Beta[(1 - Sin[c + d*x])/2, 1/2 + n, 1/2]*Sec[c + d*x]*(1 - Sin[c +
d*x])^(1/2 - n)*Sqrt[1 + Sin[c + d*x]]*(a - a*Sin[c + d*x])^n)/d)
```

3.15.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a-a\sin(c+dx))^ndx$$

$$\Big\downarrow \text{ 3131}$$

$$(1-\sin(c+dx))^{-n}(a-a\sin(c+dx))^n\int (1-\sin(c+dx))^ndx$$

$$\Big\downarrow \text{ 3042}$$

$$(1-\sin(c+dx))^{-n}(a-a\sin(c+dx))^n\int (1-\sin(c+dx))^ndx$$

$$\Big\downarrow \text{ 3130}$$

$$\frac{2^{n+\frac{1}{2}}\cos(c+dx)(1-\sin(c+dx))^{-n-\frac{1}{2}}(a-a\sin(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(\sin(c+dx)+1)\right)}{d}$$

input
```
Int[(a - a*Sin[c + d*x])^n,x]
```

output
```
(2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 + Sin[c
+ d*x])/2]*(1 - Sin[c + d*x])^(-1/2 - n)*(a - a*Sin[c + d*x])^n)/d
```

3.15.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.15.4 Maple [F]

$$\int (a - a\sin(dx + c))^n \, dx$$

input
```
int((a-a*sin(d*x+c))^n,x)
```

output
```
int((a-a*sin(d*x+c))^n,x)
```

3.15.5 Fricas [F]

$$\int (a - a\sin(c + dx))^n \, dx = \int (-a\sin(dx + c) + a)^n \, dx$$

input
```
integrate((a-a*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((-a*sin(d*x + c) + a)^n, x)
```

3.15.6 Sympy [F]

$$\int (a-a\sin(c+dx))^n\,dx=\int (-a\sin{(c+dx)}+a)^n\,dx$$

input
```
integrate((a-a*sin(d*x+c))**n,x)
```

output
```
Integral((-a*sin(c + d*x) + a)**n, x)
```

3.15.7 Maxima [F]

$$\int (a-a\sin(c+dx))^n\,dx=\int (-a\sin{(dx+c)}+a)^n\,dx$$

input
```
integrate((a-a*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((-a*sin(d*x + c) + a)^n, x)
```

3.15.8 Giac [F]

$$\int (a-a\sin(c+dx))^n\,dx=\int (-a\sin{(dx+c)}+a)^n\,dx$$

input
```
integrate((a-a*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((-a*sin(d*x + c) + a)^n, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int (a - a\sin(c + dx))^n \, dx = \int (a - a\ \sin{(c + d\,x)})^n \, dx$$

input
```
int((a - a*sin(c + d*x))^n,x)
```

output
```
int((a - a*sin(c + d*x))^n, x)
```

3.16 $\int (2+2\sin(c+dx))^n\,dx$

3.16.1 Optimal result

Integrand size = 12, antiderivative size = 60

$$\int (2+2\sin(c+dx))^n\,dx = -\frac{2^{\frac{1}{2}+2n}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{1+\sin(c+dx)}}$$

output
```
-2^(1/2+2*n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))/d
/(1+sin(d*x+c))^(1/2)
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int (2+2\sin(c+dx))^n\,dx = \frac{4^n B_{\frac{1}{2}(1+\sin(c+dx))}\left(\frac{1}{2}+n,\frac{1}{2}\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)}{d}$$

input
```
Integrate[(2 + 2*Sin[c + d*x])^n,x]
```

output
```
(4^n*Beta[(1 + Sin[c + d*x])/2, 1/2 + n, 1/2]*Sqrt[Cos[c + d*x]^2]*Sec[c +
 d*x])/d
```

3.16.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2\sin(c+dx)+2)^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (2\sin(c+dx)+2)^n dx$$

$$\Big\downarrow \text{ 3130}$$

$$-\frac{2^{2n+\frac{1}{2}}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

input
```
Int[(2 + 2*Sin[c + d*x])^n,x]
```

output
```
-((2^(1/2 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Si
n[c + d*x])/2])/(d*Sqrt[1 + Sin[c + d*x]]))
```

3.16.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

3.16.4 Maple [F]

$$\int (2 + 2\sin(dx + c))^n \, dx$$

input
```
int((2+2*sin(d*x+c))^n,x)
```

output
```
int((2+2*sin(d*x+c))^n,x)
```

3.16.5 Fricas [F]

$$\int (2 + 2\sin(c + dx))^n \, dx = \int (2 \sin(dx + c) + 2)^n \, dx$$

input
```
integrate((2+2*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*sin(d*x + c) + 2)^n, x)
```

3.16.6 Sympy [F]

$$\int (2 + 2\sin(c + dx))^n \, dx = 2^n \int (\sin(c + dx) + 1)^n \, dx$$

input
```
integrate((2+2*sin(d*x+c))**n,x)
```

output
```
2**n*Integral((sin(c + d*x) + 1)**n, x)
```

3.16.7 Maxima [F]

$$\int (2+2\sin(c+dx))^n\,dx = \int (2\sin(dx+c)+2)^n\,dx$$

input
```
integrate((2+2*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((2*sin(d*x + c) + 2)^n, x)
```

3.16.8 Giac [F]

$$\int (2+2\sin(c+dx))^n\,dx = \int (2\sin(dx+c)+2)^n\,dx$$

input
```
integrate((2+2*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((2*sin(d*x + c) + 2)^n, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int (2+2\sin(c+dx))^n\,dx = \int (2\sin(c+d\,x)+2)^n\,dx$$

input
```
int((2*sin(c + d*x) + 2)^n,x)
```

output
```
int((2*sin(c + d*x) + 2)^n, x)
```

3.17 $\int (2-2\sin(c+dx))^n\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (2-2\sin(c+dx))^n\,dx = \frac{2^{\frac{1}{2}+2n}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1+\sin(c+dx))\right)}{d\sqrt{1-\sin(c+dx)}}$$

output
```
2^(1/2+2*n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2+1/2*sin(d*x+c))/d/
(1-sin(d*x+c))^(1/2)
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int (2-2\sin(c+dx))^n\,dx = -\frac{4^n B_{\frac{1}{2}(1-\sin(c+dx))}\left(\frac{1}{2}+n,\frac{1}{2}\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)}{d}$$

input
```
Integrate[(2 - 2*Sin[c + d*x])^n,x]
```

output
```
-((4^n*Beta[(1 - Sin[c + d*x])/2, 1/2 + n, 1/2]*Sqrt[Cos[c + d*x]^2]*Sec[c
 + d*x])/d)
```

3.17.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-2\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (2-2\sin(c+dx))^ndx$$

$$\Big\downarrow \text{3130}$$

$$\frac{2^{2n+\frac{1}{2}}\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

input
```
Int[(2 - 2*Sin[c + d*x])^n,x]
```

output
```
(2^(1/2 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 + Sin[
c + d*x])/2])/(d*Sqrt[1 - Sin[c + d*x]])
```

3.17.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

3.17.4 Maple [F]

$$\int (2 - 2\sin(dx + c))^n \, dx$$

input
```
int((2-2*sin(d*x+c))^n,x)
```

output
```
int((2-2*sin(d*x+c))^n,x)
```

3.17.5 Fricas [F]

$$\int (2 - 2\sin(c + dx))^n \, dx = \int (-2 \sin(dx + c) + 2)^n \, dx$$

input
```
integrate((2-2*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((-2*sin(d*x + c) + 2)^n, x)
```

3.17.6 Sympy [F]

$$\int (2 - 2\sin(c + dx))^n \, dx = \int (2 - 2\sin(c + dx))^n \, dx$$

input
```
integrate((2-2*sin(d*x+c))**n,x)
```

output
```
Integral((2 - 2*sin(c + d*x))**n, x)
```

3.17.7 Maxima [F]

$$\int (2-2\sin(c+dx))^n\,dx=\int (-2\,\sin\left(dx+c\right)+2)^n\,dx$$

input
```
integrate((2-2*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((-2*sin(d*x + c) + 2)^n, x)
```

3.17.8 Giac [F]

$$\int (2-2\sin(c+dx))^n\,dx=\int (-2\,\sin\left(dx+c\right)+2)^n\,dx$$

input
```
integrate((2-2*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((-2*sin(d*x + c) + 2)^n, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int (2-2\sin(c+dx))^n\,dx=\int (2-2\,\sin\left(c+d\,x\right))^n\,dx$$

input
```
int((2 - 2*sin(c + d*x))^n,x)
```

output
```
int((2 - 2*sin(c + d*x))^n, x)
```

3.18 $\int \frac{1}{5+3\sin(c+dx)}\,dx$

3.18.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{5+3\sin(c+dx)}\,dx=\frac{x}{4}+\frac{\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{2d}$$

output
```
1/4*x+1/2*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d
```

3.18.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{5+3\sin(c+dx)}\,dx=\frac{\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2d}$$

input
```
Integrate[(5 + 3*Sin[c + d*x])^(-1),x]
```

output
```
ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])]/(2*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sin(c+dx)+5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{3\sin(c+dx)+5}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}+\frac{x}{4}$$

input
```
Int[(5 + 3*Sin[c + d*x])^(-1),x]
```

output
```
x/4 + ArcTan[Cos[c + d*x]/(3 + Sin[c + d*x])]/(2*d)
```

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

3.18.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{2d}$	20
default	$\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{2d}$	20
risch	$-\frac{i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i}{3}\right)}{4d}+\frac{i\ln\left(\mathrm{e}^{i(dx+c)}+3i\right)}{4d}$	40
parallelrisch	$-\frac{i\left(\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)-\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3+4i\right)\right)}{4d}$	42

input
```
int(1/(5+3*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*arctan(5/4*tan(1/2*d*x+1/2*c)+3/4)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{5+3\sin(c+dx)}\,dx=\frac{\arctan\left(\frac{5\sin(dx+c)+3}{4\cos(dx+c)}\right)}{4d}$$

input
```
integrate(1/(5+3*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c))/d
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \frac{1}{5+3\sin(c+dx)}\,dx=\begin{cases}\frac{\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0\\ \frac{x}{3\sin(c)+5} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(5+3*sin(d*x+c)),x)
```

output
```
Piecewise(((atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/
2)/pi))/(2*d), Ne(d, 0)), (x/(3*sin(c) + 5), True))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{5+3\sin(c+dx)}\,dx = \frac{\arctan\left(\frac{5\,\sin(dx+c)}{4\,(\cos(dx+c)+1)}+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) + 3/4)/d
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{1}{5+3\sin(c+dx)}\,dx = \frac{dx+c+2\,\arctan\left(-\frac{3\,\cos(dx+c)+\sin(dx+c)+3}{\cos(dx+c)-3\,\sin(dx+c)-9}\right)}{4\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(d*x + c + 2*arctan(-(3*cos(d*x + c) + sin(d*x + c) + 3)/(cos(d*x + c)
 - 3*sin(d*x + c) - 9)))/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{5+3\sin(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{3}{4}\right)}{2\,d} - \frac{\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}}{2\,d}$$

input
```
int(1/(3*sin(c + d*x) + 5),x)
```

output
```
atan((5*tan(c/2 + (d*x)/2))/4 + 3/4)/(2*d) - (atan(tan(c/2 + (d*x)/2)) - (
d*x)/2)/(2*d)
```

3.19 $\int \frac{1}{(5+3\sin(c+dx))^2}\,dx$

3.19.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(5+3\sin(c+dx))^2}\,dx = \frac{5x}{64}+\frac{5\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{32d}+\frac{3\cos(c+dx)}{16d(5+3\sin(c+dx))}$$

output
```
5/64*x+5/32*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d+3/16*cos(d*x+c)/d/(5+3*sin
(d*x+c))
```

3.19.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.62

$$\int \frac{1}{(5+3\sin(c+dx))^2}\,dx = \frac{25\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)+\frac{6(-5+5\cos(c+dx)-3\sin(c+dx))}{5+3\sin(c+dx)}}{160d}$$

input
```
Integrate[(5 + 3*Sin[c + d*x])^(-2),x]
```

output
```
(25*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])] + (6*(-5 + 5*Cos[c + d*x] - 3*Sin[c + d*x]))/(5 + 3*Sin[
c + d*x]))/(160*d)
```

3.19.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3143, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)+5)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}-\frac{1}{16}\int -\frac{5}{3\sin(c+dx)+5}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}$$

$$\Big\downarrow \text{3136}$$

$$\frac{5}{16}\left(\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}+\frac{x}{4}\right)+\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}$$

input
```
Int[(5 + 3*Sin[c + d*x])^(-2),x]
```

output
```
(5*(x/4 + ArcTan[Cos[c + d*x]/(3 + Sin[c + d*x])]/(2*d)))/16 + (3*Cos[c +
d*x])/(16*d*(5 + 3*Sin[c + d*x]))
```

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.19.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}+\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{32}}{d}$
default	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}+\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{32}}{d}$
risch	$\frac{5\,e^{i(dx+c)}+3i}{8d\left(3\,e^{2i(dx+c)}-3+10i e^{i(dx+c)}\right)}+\frac{5i\ln\left(e^{i(dx+c)}+3i\right)}{64d}-\frac{5i\ln\left(e^{i(dx+c)}+\frac{i}{3}\right)}{64d}$
parallelrisch	$\frac{(-75i\sin(dx+c)-125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)+(75i\sin(dx+c)+125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3+4i\right)+60\cos(dx+c)+36}{960d\sin(dx+c)+1600d}$

input
```
int(1/(5+3*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(9/400*tan(1/2*d*x+1/2*c)+3/80)/(tan(1/2*d*x+1/2*c)^2+6/5*tan(1/2*d
*x+1/2*c)+1)+5/32*arctan(5/4*tan(1/2*d*x+1/2*c)+3/4))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(5+3\sin(c+dx))^2}\,dx = \frac{5\,(3\,\sin\left(dx+c\right)+5)\arctan\left(\frac{5\,\sin(dx+c)+3}{4\,\cos(dx+c)}\right)+12\,\cos\left(dx+c\right)}{64\,(3\,d\sin\left(dx+c\right)+5\,d)}$$

input
```
integrate(1/(5+3*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/64*(5*(3*sin(d*x + c) + 5)*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c))
 + 12*cos(d*x + c))/(3*d*sin(d*x + c) + 5*d)
```

3.19.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 388, normalized size of antiderivative = 6.93

$$\int \frac{1}{(5+3\sin(c+dx))^2}\,dx = \begin{cases} \frac{x}{\left(5-3\sin\left(2\operatorname{atan}\left(\frac{3}{5}-\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{\left(5-3\sin\left(2\operatorname{atan}\left(\frac{3}{5}+\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{(3\sin(c)+5)^2} \\ \frac{125\left(\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} + \frac{150\left(\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} + \frac{125(}{800} \end{cases}$$

input
```
integrate(1/(5+3*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(5 - 3*sin(2*atan(3/5 - 4*I/5)))**2, Eq(c, -d*x - 2*atan(3/5
- 4*I/5))), (x/(5 - 3*sin(2*atan(3/5 + 4*I/5)))**2, Eq(c, -d*x - 2*atan(3/
5 + 4*I/5))), (x/(3*sin(c) + 5)**2, Eq(d, 0)), (125*(atan(5*tan(c/2 + d*x/
2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(800*
d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 150*(atan(5*tan(
c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2
)/(800*d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 125*(atan
(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(800*d*t
an(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 36*tan(c/2 + d*x/2)
/(800*d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 60/(800*d*
tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d), True))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \frac{1}{(5+3\sin(c+dx))^2}\,dx = \frac{\frac{12\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+5\right)}{\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5} + 25\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}+\frac{3}{4}\right)}{160\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/160*(12*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5)/(6*sin(d*x + c)/(cos(d*x
 + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5) + 25*arctan(5/4*si
n(d*x + c)/(cos(d*x + c) + 1) + 3/4))/d
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{1}{(5+3\sin(c+dx))^2}\,dx = \frac{25\,dx+25\,c+\frac{24\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}+50\arctan\left(-\frac{3\cos(dx+c)+\sin(dx+c)+3}{\cos(dx+c)-3\sin(dx+c)-9}\right)}{320\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/320*(25*d*x + 25*c + 24*(3*tan(1/2*d*x + 1/2*c) + 5)/(5*tan(1/2*d*x + 1/
2*c)^2 + 6*tan(1/2*d*x + 1/2*c) + 5) + 50*arctan(-(3*cos(d*x + c) + sin(d*
x + c) + 3)/(cos(d*x + c) - 3*sin(d*x + c) - 9)))/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.48

$$\int \frac{1}{(5+3\sin(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3}{4}\right)}{32\,d} - \frac{5\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{32\,d} + \frac{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{200}+\frac{3}{40}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+1\right)}$$

input
```
int(1/(3*sin(c + d*x) + 5)^2,x)
```

output
```
(5*atan((5*tan(c/2 + (d*x)/2))/4 + 3/4))/(32*d) - (5*(atan(tan(c/2 + (d*x)
/2)) - (d*x)/2))/(32*d) + ((9*tan(c/2 + (d*x)/2))/200 + 3/40)/(d*((6*tan(c
/2 + (d*x)/2))/5 + tan(c/2 + (d*x)/2)^2 + 1))
```

3.20 $\int \frac{1}{(5+3\sin(c+dx))^3}\,dx$

3.20.1 Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(5+3\sin(c+dx))^3}\,dx = \frac{59x}{2048} + \frac{59\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{1024d} + \frac{3\cos(c+dx)}{32d(5+3\sin(c+dx))^2} + \frac{45\cos(c+dx)}{512d(5+3\sin(c+dx))}$$

output
```
59/2048*x+59/1024*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d+3/32*cos(d*x+c)/d/(5
+3*sin(d*x+c))^2+45/512*cos(d*x+c)/d/(5+3*sin(d*x+c))
```

3.20.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{1}{(5+3\sin(c+dx))^3}\,dx = \frac{59\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{546\cos(c+dx)+9(-59+9\cos(2(c+dx))-60\sin(c+dx)+15\sin(2(c+dx)))}{(5+3\sin(c+dx))^2}}{1024d}$$

input
```
Integrate[(5 + 3*Sin[c + d*x])^(-3),x]
```

output
```
(59*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])] + (546*Cos[c + d*x] + 9*(-59 + 9*Cos[2*(c + d*x)] - 60*S
in[c + d*x] + 15*Sin[2*(c + d*x)]))/(5 + 3*Sin[c + d*x])^2)/(1024*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(3\sin(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{ 3143}$$

$$\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}-\frac{1}{32}\int -\frac{10-3\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{32}\int \frac{10-3\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx+\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{32}\int \frac{10-3\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx+\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3233}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}-\frac{1}{16}\int -\frac{59}{3\sin(c+dx)+5}dx\right)+\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3136}$$

$$\frac{1}{32}\left(\frac{59}{16}\left(\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}+\frac{x}{4}\right)+\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

input
```
Int[(5 + 3*Sin[c + d*x])^(-3),x]
```

output
```
(3*Cos[c + d*x])/(32*d*(5 + 3*Sin[c + d*x])^2) + ((59*(x/4 + ArcTan[Cos[c
+ d*x]/(3 + Sin[c + d*x])]/(2*d)))/16 + (45*Cos[c + d*x])/(16*d*(5 + 3*Sin
[c + d*x])))/32
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.20.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}+\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6400}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1280}+\frac{273}{256}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}+\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{1024}}{d}$
default	$\frac{\frac{\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}+\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6400}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1280}+\frac{273}{256}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}+\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{1024}}{d}$
risch	$\frac{\frac{885ie^{2i(dx+c)}}{256}+\frac{177\,e^{3i(dx+c)}}{256}-\frac{723\,e^{i(dx+c)}}{256}-\frac{135i}{256}}{\left(3\,e^{2i(dx+c)}-3+10ie^{i(dx+c)}\right)^2 d}+\frac{59i\ln\left(e^{i(dx+c)}+3i\right)}{2048d}-\frac{59i\ln\left(e^{i(dx+c)}+\frac{i}{3}\right)}{2048d}$
parallelrisch	$\frac{-1062+59i(59-9\cos(2dx+2c)+60\sin(dx+c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)+59i(-59+9\cos(2dx+2c)-60\sin(dx+c))\ln\left(5\right.}{2048d(-59+9\cos(2dx+2c)-60\sin}$

input
```
int(1/(5+3*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(50*(963/64000*tan(1/2*d*x+1/2*c)^3+11739/320000*tan(1/2*d*x+1/2*c)^2+
2313/64000*tan(1/2*d*x+1/2*c)+273/12800)/(5*tan(1/2*d*x+1/2*c)^2+6*tan(1/2
*d*x+1/2*c)+5)^2+59/1024*arctan(5/4*tan(1/2*d*x+1/2*c)+3/4))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{1}{(5+3\sin(c+dx))^3}\,dx$$
$$= \frac{59\left(9\cos\left(dx+c\right)^2-30\sin\left(dx+c\right)-34\right)\arctan\left(\frac{5\sin(dx+c)+3}{4\cos(dx+c)}\right)-540\cos\left(dx+c\right)\sin\left(dx+c\right)-1092\ c}{2048\left(9\,d\cos\left(dx+c\right)^2-30\,d\sin\left(dx+c\right)-34\,d\right)}$$

input
```
integrate(1/(5+3*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2048*(59*(9*cos(d*x + c)^2 - 30*sin(d*x + c) - 34)*arctan(1/4*(5*sin(d*x
 + c) + 3)/cos(d*x + c)) - 540*cos(d*x + c)*sin(d*x + c) - 1092*cos(d*x +
c))/(9*d*cos(d*x + c)^2 - 30*d*sin(d*x + c) - 34*d)
```

3.20.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 918, normalized size of antiderivative = 11.33

$$\int \frac{1}{(5+3\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(5 - 3*sin(2*atan(3/5 - 4*I/5)))**3, Eq(c, -d*x - 2*atan(3/5
- 4*I/5))), (x/(5 - 3*sin(2*atan(3/5 + 4*I/5)))**3, Eq(c, -d*x - 2*atan(3/
5 + 4*I/5))), (x/(3*sin(c) + 5)**3, Eq(d, 0)), (36875*(atan(5*tan(c/2 + d*
x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(64
0000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c/2 + d*x/2)**3 + 2201600*d*tan
(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2) + 640000*d) + 88500*(atan(5*
tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d
*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c/2 + d*x/2)**3 + 2
201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2) + 640000*d) + 12
6850*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi)
)*tan(c/2 + d*x/2)**2/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c/2 +
d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2) + 6
40000*d) + 88500*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2
 - pi/2)/pi))*tan(c/2 + d*x/2)/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*t
an(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d
*x/2) + 640000*d) + 36875*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/
2 + d*x/2 - pi/2)/pi))/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c/2 +
 d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2) +
640000*d) + 19260*tan(c/2 + d*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 + 1536
000*d*tan(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*t...
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.14

$$\int \frac{1}{(5+3\sin(c+dx))^3}\,dx = \frac{\frac{12\left(\frac{3855\sin(dx+c)}{\cos(dx+c)+1}+\frac{3913\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1605\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+2275\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1}+\frac{86\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+25}+1475\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}+\frac{3}{4}\right)}{25600\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/25600*(12*(3855*sin(d*x + c)/(cos(d*x + c) + 1) + 3913*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1605*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2275)/(60*
sin(d*x + c)/(cos(d*x + c) + 1) + 86*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 60*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 25*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 25) + 1475*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) + 3/4))/d
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.49

$$\int \frac{1}{(5+3\sin(c+dx))^3}\,dx = \frac{1475\,dx + 1475\,c + \frac{24\left(1605\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3913\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2275\right)}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^2} + 2950\,\arctan\left(-\frac{3\,\cos(dx+c)}{\cos(dx+c)-}\right.}{51200\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/51200*(1475*d*x + 1475*c + 24*(1605*tan(1/2*d*x + 1/2*c)^3 + 3913*tan(1/
2*d*x + 1/2*c)^2 + 3855*tan(1/2*d*x + 1/2*c) + 2275)/(5*tan(1/2*d*x + 1/2*
c)^2 + 6*tan(1/2*d*x + 1/2*c) + 5)^2 + 2950*arctan(-(3*cos(d*x + c) + sin(
d*x + c) + 3)/(cos(d*x + c) - 3*sin(d*x + c) - 9)))/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5+3\sin(c+dx))^3}\,dx = \frac{59\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3}{4}\right)}{1024\,d} - \frac{59\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d} + \frac{\frac{963\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1280}+\frac{11739\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400}+\frac{2313\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1280}+\frac{273}{256}}{d\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\right)^2}$$

input
```
int(1/(3*sin(c + d*x) + 5)^3,x)
```

output
```
(59*atan((5*tan(c/2 + (d*x)/2))/4 + 3/4))/(1024*d) - (59*(atan(tan(c/2 + (
d*x)/2)) - (d*x)/2))/(1024*d) + ((2313*tan(c/2 + (d*x)/2))/1280 + (11739*t
an(c/2 + (d*x)/2)^2)/6400 + (963*tan(c/2 + (d*x)/2)^3)/1280 + 273/256)/(d*
(6*tan(c/2 + (d*x)/2) + 5*tan(c/2 + (d*x)/2)^2 + 5)^2)
```

3.20. $\int \frac{1}{(5+3\sin(c+dx))^3}\,dx$

3.21 $\int \frac{1}{(5+3\sin(c+dx))^4}\,dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(5+3\sin(c+dx))^4}\,dx = \frac{385x}{32768} + \frac{385\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{16384d} + \frac{\cos(c+dx)}{16d(5+3\sin(c+dx))^3} + \frac{25\cos(c+dx)}{512d(5+3\sin(c+dx))^2} + \frac{311\cos(c+dx)}{8192d(5+3\sin(c+dx))}$$

output
```
385/32768*x+385/16384*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d+1/16*cos(d*x+c)/
d/(5+3*sin(d*x+c))^3+25/512*cos(d*x+c)/d/(5+3*sin(d*x+c))^2+311/8192*cos(d
*x+c)/d/(5+3*sin(d*x+c))
```

3.21.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int \frac{1}{(5+3\sin(c+dx))^4}\,dx = \frac{1925\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{-239470+219735\cos(c+dx)+83970\cos(2(c+dx))-13995\cos(3(c+dx))-305091\sin(c}{2(5+3\sin(c+dx))^3}}{81920d}$$

input
```
Integrate[(5 + 3*Sin[c + d*x])^(-4),x]
```

output
```
(1925*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])] + (-239470 + 219735*Cos[c + d*x] + 83970*Cos[2*(c + d*
x)] - 13995*Cos[3*(c + d*x)] - 305091*Sin[c + d*x] + 105300*Sin[2*(c + d*x
)] + 8397*Sin[3*(c + d*x)])/(2*(5 + 3*Sin[c + d*x])^3))/(81920*d)
```

3.21.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3143, 27, 3042, 3233, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)+5)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(c+dx)+5)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}-\frac{1}{48}\int -\frac{3(5-2\sin(c+dx))}{(3\sin(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{5-2\sin(c+dx)}{(3\sin(c+dx)+5)^3}dx+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \frac{5-2\sin(c+dx)}{(3\sin(c+dx)+5)^3}dx+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}-\frac{1}{32}\int -\frac{62-25\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}-\frac{1}{16}\int -\frac{385}{3\sin(c+dx)+5}dx\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\left(\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}+\frac{x}{4}\right)+\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

input
```
Int[(5 + 3*Sin[c + d*x])^(-4),x]
```

output
```
Cos[c + d*x]/(16*d*(5 + 3*Sin[c + d*x])^3) + ((25*Cos[c + d*x])/(32*d*(5 +
 3*Sin[c + d*x])^2) + ((385*(x/4 + ArcTan[Cos[c + d*x]/(3 + Sin[c + d*x])]
/(2*d)))/16 + (311*Cos[c + d*x])/(16*d*(5 + 3*Sin[c + d*x])))/32)/16
```

3.21. $\int \frac{1}{(5+3\sin(c+dx))^4}\,dx$

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.21.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}+\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}+\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}+\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
default	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}+\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}+\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}+\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
risch	$\frac{-239470\,e^{3i(dx+c)}+86625ie^{4i(dx+c)}-218466ie^{2i(dx+c)}+10395\,e^{5i(dx+c)}+73575\,e^{i(dx+c)}+8397i}{12288\left(3\,e^{2i(dx+c)}-3+10ie^{i(dx+c)}\right)^3d}-\frac{385i\ln\left(e^{i(dx+c)}+\right.}{32768d}$
parallelrisch	$\frac{-31683960+48125i(770-27\sin(3dx+3c)+981\sin(dx+c)-270\cos(2dx+2c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)+48125i(27\sin(}{}$

input
```
int(1/(5+3*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(250*(39933/5120000*tan(1/2*d*x+1/2*c)^5+672723/25600000*tan(1/2*d*x+1
/2*c)^4+2870073/64000000*tan(1/2*d*x+1/2*c)^3+604899/12800000*tan(1/2*d*x+
1/2*c)^2+145233/5120000*tan(1/2*d*x+1/2*c)+10287/1024000)/(5*tan(1/2*d*x+1
/2*c)^2+6*tan(1/2*d*x+1/2*c)+5)^3+385/16384*arctan(5/4*tan(1/2*d*x+1/2*c)+
3/4))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{1}{(5+3\sin(c+dx))^4}\,dx$$
$$=\frac{11196\cos(dx+c)^3+385\left(135\cos(dx+c)^2+9\left(3\cos(dx+c)^2-28\right)\sin(dx+c)-260\right)\arctan\left(\frac{5\sin(\text{}}{4\cos}\right.}{32768\left(135\,d\cos(dx+c)^2+9\left(3\,d\cos(dx+c)^2-28\,d\right)\sin\right.}$$

input
```
integrate(1/(5+3*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/32768*(11196*cos(d*x + c)^3 + 385*(135*cos(d*x + c)^2 + 9*(3*cos(d*x + c
)^2 - 28)*sin(d*x + c) - 260)*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c)
) - 42120*cos(d*x + c)*sin(d*x + c) - 52344*cos(d*x + c))/(135*d*cos(d*x +
 c)^2 + 9*(3*d*cos(d*x + c)^2 - 28*d)*sin(d*x + c) - 260*d)
```

3.21.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.35 (sec) , antiderivative size = 1693, normalized size of antiderivative = 15.97

$$\int \frac{1}{(5+3\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(5 - 3*sin(2*atan(3/5 - 4*I/5)))**4, Eq(c, -d*x - 2*atan(3/5
- 4*I/5))), (x/(5 - 3*sin(2*atan(3/5 + 4*I/5)))**4, Eq(c, -d*x - 2*atan(3/
5 + 4*I/5))), (x/(3*sin(c) + 5)**4, Eq(d, 0)), (6015625*(atan(5*tan(c/2 +
d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(
256000000*d*tan(c/2 + d*x/2)**6 + 921600000*d*tan(c/2 + d*x/2)**5 + 187392
0000*d*tan(c/2 + d*x/2)**4 + 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000
*d*tan(c/2 + d*x/2)**2 + 921600000*d*tan(c/2 + d*x/2) + 256000000*d) + 216
56250*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi
))*tan(c/2 + d*x/2)**5/(256000000*d*tan(c/2 + d*x/2)**6 + 921600000*d*tan(
c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)**4 + 2285568000*d*tan(c/2
+ d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2 + 921600000*d*tan(c/2 + d*x
/2) + 256000000*d) + 44034375*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor
((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(256000000*d*tan(c/2 + d*x/
2)**6 + 921600000*d*tan(c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)**4
 + 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2 + 9
21600000*d*tan(c/2 + d*x/2) + 256000000*d) + 53707500*(atan(5*tan(c/2 + d*
x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**3/(25
6000000*d*tan(c/2 + d*x/2)**6 + 921600000*d*tan(c/2 + d*x/2)**5 + 18739200
00*d*tan(c/2 + d*x/2)**4 + 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000*d
*tan(c/2 + d*x/2)**2 + 921600000*d*tan(c/2 + d*x/2) + 256000000*d) + 44...
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(96) = 192.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.39

$$\int \frac{1}{(5+3\sin(c+dx))^4}\,dx$$
$$= \frac{\frac{36\left(\frac{403425\sin(dx+c)}{\cos(dx+c)+1}+\frac{672110\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{637794\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{373735\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{110925\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+142875\right)}{\frac{450\sin(dx+c)}{\cos(dx+c)+1}+\frac{915\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1116\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{915\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{450\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{125\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+125}+48125\arctan\left(\frac{5\sin(dx\cdots}{4(\cos(dx+\cdots}\right.}{2048000\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2048000*(36*(403425*sin(d*x + c)/(cos(d*x + c) + 1) + 672110*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 637794*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 37
3735*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 110925*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 142875)/(450*sin(d*x + c)/(cos(d*x + c) + 1) + 915*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 1116*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 91
5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 450*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 + 125*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 125) + 48125*arctan(5/4*
sin(d*x + c)/(cos(d*x + c) + 1) + 3/4))/d
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{1}{(5+3\sin(c+dx))^4}\,dx$$
$$= \frac{48125\,dx + 48125\,c + \frac{72\left(110925\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+373735\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+637794\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+672110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+403425}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^3}}{4096000\,d}$$

input
```
integrate(1/(5+3*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/4096000*(48125*d*x + 48125*c + 72*(110925*tan(1/2*d*x + 1/2*c)^5 + 37373
5*tan(1/2*d*x + 1/2*c)^4 + 637794*tan(1/2*d*x + 1/2*c)^3 + 672110*tan(1/2*
d*x + 1/2*c)^2 + 403425*tan(1/2*d*x + 1/2*c) + 142875)/(5*tan(1/2*d*x + 1/
2*c)^2 + 6*tan(1/2*d*x + 1/2*c) + 5)^3 + 96250*arctan(-(3*cos(d*x + c) + s
in(d*x + c) + 3)/(cos(d*x + c) - 3*sin(d*x + c) - 9)))/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.76

$$\int \frac{1}{(5+3\sin(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3}{4}\right)}{16384\,d} - \frac{385\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d}$$
$$+ \frac{\frac{39933\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2560000}+\frac{672723\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{12800000}+\frac{2870073\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000000}+\frac{604899\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400000}+\frac{145233\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2560000}+\frac{10287}{512000}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{25}+\frac{1116\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{125}+\frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{25}+\frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+1\right)}$$

input
```
int(1/(3*sin(c + d*x) + 5)^4,x)
```

output
```
(385*atan((5*tan(c/2 + (d*x)/2))/4 + 3/4))/(16384*d) - (385*(atan(tan(c/2
+ (d*x)/2)) - (d*x)/2))/(16384*d) + ((145233*tan(c/2 + (d*x)/2))/2560000 +
 (604899*tan(c/2 + (d*x)/2)^2)/6400000 + (2870073*tan(c/2 + (d*x)/2)^3)/32
000000 + (672723*tan(c/2 + (d*x)/2)^4)/12800000 + (39933*tan(c/2 + (d*x)/2
)^5)/2560000 + 10287/512000)/(d*((18*tan(c/2 + (d*x)/2))/5 + (183*tan(c/2
+ (d*x)/2)^2)/25 + (1116*tan(c/2 + (d*x)/2)^3)/125 + (183*tan(c/2 + (d*x)/
2)^4)/25 + (18*tan(c/2 + (d*x)/2)^5)/5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.21. $\int \frac{1}{(5+3\sin(c+dx))^4}\,dx$

3.22 $\int \frac{1}{5-3\sin(c+dx)}\,dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{5-3\sin(c+dx)}\,dx = \frac{x}{4} - \frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}$$

output
```
1/4*x-1/2*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{1}{5-3\sin(c+dx)}\,dx = -\frac{\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2d}$$

input
```
Integrate[(5 - 3*Sin[c + d*x])^(-1),x]
```

output
```
-1/2*ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])]/d
```

3.22.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5-3\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{5-3\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3136}$$

$$\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}$$

input
```
Int[(5 - 3*Sin[c + d*x])^(-1),x]
```

output
```
x/4 - ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

3.22.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{2d}$	20
default	$\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{2d}$	20
risch	$\frac{i\ln\left(\mathrm{e}^{i(dx+c)}-3i\right)}{4d}-\frac{i\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i}{3}\right)}{4d}$	40
parallelrisch	$-\frac{i\left(\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)-\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3+4i\right)\right)}{4d}$	42

input
```
int(1/(5-3*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*arctan(5/4*tan(1/2*d*x+1/2*c)-3/4)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{5-3\sin(c+dx)}\,dx = \frac{\arctan\left(\frac{5\sin(dx+c)-3}{4\cos(dx+c)}\right)}{4\,d}$$

input
```
integrate(1/(5-3*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*arctan(1/4*(5*sin(d*x + c) - 3)/cos(d*x + c))/d
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{1}{5-3\sin(c+dx)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0 \\ \frac{x}{5-3\sin(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(5-3*sin(d*x+c)),x)
```

output
```
Piecewise(((atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/
2)/pi))/(2*d), Ne(d, 0)), (x/(5 - 3*sin(c)), True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{5-3\sin(c+dx)}\,dx = \frac{\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}-\frac{3}{4}\right)}{2d}$$

input
```
integrate(1/(5-3*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) - 3/4)/d
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{1}{5-3\sin(c+dx)}\,dx = \frac{dx+c+2\arctan\left(\frac{3\cos(dx+c)-\sin(dx+c)+3}{\cos(dx+c)+3\sin(dx+c)-9}\right)}{4d}$$

input
```
integrate(1/(5-3*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(d*x + c + 2*arctan((3*cos(d*x + c) - sin(d*x + c) + 3)/(cos(d*x + c)
+ 3*sin(d*x + c) - 9)))/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{5-3\sin(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)}{2\,d} - \frac{\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}}{2\,d}$$

input
```
int(-1/(3*sin(c + d*x) - 5),x)
```

output
```
atan((5*tan(c/2 + (d*x)/2))/4 - 3/4)/(2*d) - (atan(tan(c/2 + (d*x)/2)) - (
d*x)/2)/(2*d)
```

3.23 $\int \frac{1}{(5-3\sin(c+dx))^2} dx$

3.23.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(5-3\sin(c+dx))^2} dx = \frac{5x}{64} - \frac{5\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{32d} - \frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$

output
```
5/64*x-5/32*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d-3/16*cos(d*x+c)/d/(5-3*sin
(d*x+c))
```

3.23.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.57

$$\int \frac{1}{(5-3\sin(c+dx))^2} dx = \frac{-25\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{6(-5+5\cos(c+dx)+3\sin(c+dx))}{-5+3\sin(c+dx)}}{160d}$$

input
```
Integrate[(5 - 3*Sin[c + d*x])^(-2),x]
```

output
```
(-25*ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])] + (6*(-5 + 5*Cos[c + d*x] + 3*Sin[c + d*x]))/(-5 + 3*Si
n[c + d*x]))/(160*d)
```

3.23.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3143, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5-3\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5-3\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{16}\int -\frac{5}{5-3\sin(c+dx)}dx-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$

$$\Big\downarrow \text{3136}$$

$$\frac{5}{16}\left(\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}\right)-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$

input
```
Int[(5 - 3*Sin[c + d*x])^(-2),x]
```

output
```
(5*(x/4 - ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)))/16 - (3*Cos[c +
d*x])/(16*d*(5 - 3*Sin[c + d*x]))
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.23.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}-\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{32}}{d}$
default	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}-\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{32}}{d}$
risch	$-\frac{5\,e^{i(dx+c)}-3i}{8d\left(3\,e^{2i(dx+c)}-3-10i\,e^{i(dx+c)}\right)}+\frac{5i\ln\left(e^{i(dx+c)}-3i\right)}{64d}-\frac{5i\ln\left(e^{i(dx+c)}-\frac{i}{3}\right)}{64d}$
parallelrisch	$\frac{(-75i\sin(dx+c)+125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)+(75i\sin(dx+c)-125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3+4i\right)+60\cos(dx+c)-36}{960d\sin(dx+c)-1600d}$

input
```
int(1/(5-3*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(9/400*tan(1/2*d*x+1/2*c)-3/80)/(tan(1/2*d*x+1/2*c)^2-6/5*tan(1/2*d
*x+1/2*c)+1)+5/32*arctan(5/4*tan(1/2*d*x+1/2*c)-3/4))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{1}{(5-3\sin(c+dx))^2}\,dx = \frac{5\,(3\,\sin\left(dx+c\right)-5)\arctan\left(\frac{5\,\sin(dx+c)-3}{4\,\cos(dx+c)}\right)+12\,\cos\left(dx+c\right)}{64\,(3\,d\sin\left(dx+c\right)-5\,d)}$$

input
```
integrate(1/(5-3*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/64*(5*(3*sin(d*x + c) - 5)*arctan(1/4*(5*sin(d*x + c) - 3)/cos(d*x + c))
 + 12*cos(d*x + c))/(3*d*sin(d*x + c) - 5*d)
```

3.23.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.62

$$\int \frac{1}{(5-3\sin(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{x}{\left(5-3\sin\left(2\operatorname{atan}\left(\frac{3}{5}-\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{\left(5-3\sin\left(2\operatorname{atan}\left(\frac{3}{5}+\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{\left(5-3\sin\left(c\right)\right)^2} \\ \frac{125\left(\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} - \frac{150\left(\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} + \frac{125(}{800} \end{cases}$$

input
```
integrate(1/(5-3*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(5 - 3*sin(2*atan(3/5 - 4*I/5)))**2, Eq(c, -d*x + 2*atan(3/5
- 4*I/5))), (x/(5 - 3*sin(2*atan(3/5 + 4*I/5)))**2, Eq(c, -d*x + 2*atan(3/
5 + 4*I/5))), (x/(5 - 3*sin(c))**2, Eq(d, 0)), (125*(atan(5*tan(c/2 + d*x/
2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(800*
d*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) - 150*(atan(5*tan(
c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2
)/(800*d*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) + 125*(atan
(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(800*d*t
an(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) + 36*tan(c/2 + d*x/2)
/(800*d*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) - 60/(800*d*
tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d), True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

$$\int \frac{1}{(5-3\sin(c+dx))^2}\,dx = -\frac{\frac{12\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-5\right)}{\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-5} - 25\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}-\frac{3}{4}\right)}{160\,d}$$

input
```
integrate(1/(5-3*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/160*(12*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5)/(6*sin(d*x + c)/(cos(d*
x + c) + 1) - 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5) - 25*arctan(5/4*s
in(d*x + c)/(cos(d*x + c) + 1) - 3/4))/d
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

$$\int \frac{1}{(5-3\sin(c+dx))^2}\,dx = \frac{25\,dx + 25\,c + \frac{24\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\right)}{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5} + 50\arctan\left(\frac{3\cos(dx+c)-\sin(dx+c)+3}{\cos(dx+c)+3\sin(dx+c)-9}\right)}{320\,d}$$

input
```
integrate(1/(5-3*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/320*(25*d*x + 25*c + 24*(3*tan(1/2*d*x + 1/2*c) - 5)/(5*tan(1/2*d*x + 1/
2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 5) + 50*arctan((3*cos(d*x + c) - sin(d*x
 + c) + 3)/(cos(d*x + c) + 3*sin(d*x + c) - 9)))/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

$$\int \frac{1}{(5-3\sin(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{3}{4}\right)}{32\,d}-\frac{5\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{32\,d}+\frac{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{200}-\frac{3}{40}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+1\right)}$$

input
```
int(1/(3*sin(c + d*x) - 5)^2,x)
```

output
```
(5*atan((5*tan(c/2 + (d*x)/2))/4 - 3/4))/(32*d) - (5*(atan(tan(c/2 + (d*x)
/2)) - (d*x)/2))/(32*d) + ((9*tan(c/2 + (d*x)/2))/200 - 3/40)/(d*(tan(c/2
+ (d*x)/2)^2 - (6*tan(c/2 + (d*x)/2))/5 + 1))
```

3.24 $\int \frac{1}{(5-3\sin(c+dx))^3}\,dx$

3.24.1 Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \frac{1}{(5-3\sin(c+dx))^3}\,dx = \frac{59x}{2048} - \frac{59\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{1024d} - \frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2} - \frac{45\cos(c+dx)}{512d(5-3\sin(c+dx))}$$

output
```
59/2048*x-59/1024*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d-3/32*cos(d*x+c)/d/(5
-3*sin(d*x+c))^2-45/512*cos(d*x+c)/d/(5-3*sin(d*x+c))
```

3.24.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \frac{1}{(5-3\sin(c+dx))^3}\,dx = -\frac{59\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{546\cos(c+dx)+9(-59+9\cos(2(c+dx))+60\sin(c+dx)-15\sin(2(c+dx)))}{(5-3\sin(c+dx))^2}}{1024d}$$

input
```
Integrate[(5 - 3*Sin[c + d*x])^(-3),x]
```

output
```
-1/1024*(59*ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])] + (546*Cos[c + d*x] + 9*(-59 + 9*Cos[2*(c + d*x)
] + 60*Sin[c + d*x] - 15*Sin[2*(c + d*x)]))/(5 - 3*Sin[c + d*x])^2)/d
```

3.24.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5-3\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5-3\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{32}\int -\frac{3\sin(c+dx)+10}{(5-3\sin(c+dx))^2}dx-\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{32}\int \frac{3\sin(c+dx)+10}{(5-3\sin(c+dx))^2}dx-\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \frac{3\sin(c+dx)+10}{(5-3\sin(c+dx))^2}dx-\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{59}{5-3\sin(c+dx)}dx-\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{32}\left(\frac{59}{16}\left(\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}\right)-\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

input
```
Int[(5 - 3*Sin[c + d*x])^(-3),x]
```

output
```
((59*(x/4 - ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)))/16 - (45*Cos[c
 + d*x])/(16*d*(5 - 3*Sin[c + d*x])))/32 - (3*Cos[c + d*x])/(32*d*(5 - 3*S
in[c + d*x])^2)
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.24.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}-\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6400}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1280}-\frac{273}{256}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}+\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{1024}}{d}$
default	$\frac{\frac{\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}-\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6400}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1280}-\frac{273}{256}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}+\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{1024}}{d}$
risch	$-\frac{3\left(-295i\,e^{2i(dx+c)}+59\,e^{3i(dx+c)}-241\,e^{i(dx+c)}+45i\right)}{256\left(3\,e^{2i(dx+c)}-3-10i\,e^{i(dx+c)}\right)^2 d}+\frac{59i\ln\left(e^{i(dx+c)}-3i\right)}{2048d}-\frac{59i\ln\left(e^{i(dx+c)}-\frac{i}{3}\right)}{2048d}$
parallelrisch	$\frac{1062+59i(59-9\cos(2dx+2c)-60\sin(dx+c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)+59i(-59+9\cos(2dx+2c)+60\sin(dx+c))\ln\left(5\,\mathrm{t}\right.}{2048d(-59+9\cos(2dx+2c)+60\sin(}$

input
```
int(1/(5-3*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(50*(963/64000*tan(1/2*d*x+1/2*c)^3-11739/320000*tan(1/2*d*x+1/2*c)^2+
2313/64000*tan(1/2*d*x+1/2*c)-273/12800)/(5*tan(1/2*d*x+1/2*c)^2-6*tan(1/2
*d*x+1/2*c)+5)^2+59/1024*arctan(5/4*tan(1/2*d*x+1/2*c)-3/4))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{1}{(5-3\sin(c+dx))^3}\,dx$$
$$=\frac{59\left(9\cos\left(dx+c\right)^2+30\sin\left(dx+c\right)-34\right)\arctan\left(\frac{5\sin(dx+c)-3}{4\cos(dx+c)}\right)-540\cos\left(dx+c\right)\sin\left(dx+c\right)+1092\,\mathrm{c}}{2048\left(9\,d\cos\left(dx+c\right)^2+30\,d\sin\left(dx+c\right)-34\,d\right)}$$

input
```
integrate(1/(5-3*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2048*(59*(9*cos(d*x + c)^2 + 30*sin(d*x + c) - 34)*arctan(1/4*(5*sin(d*x
 + c) - 3)/cos(d*x + c)) - 540*cos(d*x + c)*sin(d*x + c) + 1092*cos(d*x +
c))/(9*d*cos(d*x + c)^2 + 30*d*sin(d*x + c) - 34*d)
```

3.24.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 915, normalized size of antiderivative = 11.02

$$\int \frac{1}{(5-3\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5-3*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(5 - 3*sin(2*atan(3/5 - 4*I/5)))**3, Eq(c, -d*x + 2*atan(3/5
- 4*I/5))), (x/(5 - 3*sin(2*atan(3/5 + 4*I/5)))**3, Eq(c, -d*x + 2*atan(3/
5 + 4*I/5))), (x/(5 - 3*sin(c))**3, Eq(d, 0)), (36875*(atan(5*tan(c/2 + d*
x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(64
0000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/2 + d*x/2)**3 + 2201600*d*tan
(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2) + 640000*d) - 88500*(atan(5*
tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d
*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/2 + d*x/2)**3 + 2
201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2) + 640000*d) + 12
6850*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi)
)*tan(c/2 + d*x/2)**2/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/2 +
d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2) + 6
40000*d) - 88500*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2
 - pi/2)/pi))*tan(c/2 + d*x/2)/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*t
an(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d
*x/2) + 640000*d) + 36875*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/
2 + d*x/2 - pi/2)/pi))/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/2 +
 d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2) +
640000*d) + 19260*tan(c/2 + d*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 - 1536
000*d*tan(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*t...
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.08

$$\int \frac{1}{(5-3\sin(c+dx))^3}\,dx$$
$$= -\frac{\frac{12\left(\frac{3855\sin(dx+c)}{\cos(dx+c)+1} - \frac{3913\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{1605\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - 2275\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1} - \frac{86\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 25} - 1475\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)} - \frac{3}{4}\right)}{25600\,d}$$

input
```
integrate(1/(5-3*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/25600*(12*(3855*sin(d*x + c)/(cos(d*x + c) + 1) - 3913*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 1605*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2275)/(60
*sin(d*x + c)/(cos(d*x + c) + 1) - 86*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 60*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 25) - 1475*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) - 3/4))/d
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int \frac{1}{(5-3\sin(c+dx))^3}\,dx = \frac{1475\,dx+1475\,c+\frac{24\left(1605\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3913\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2275\right)}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^2}+2950\,\arctan\left(\frac{3\,\cos(dx+c)-}{\cos(dx+c)+3}\right.}{51200\,d}$$

input
```
integrate(1/(5-3*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/51200*(1475*d*x + 1475*c + 24*(1605*tan(1/2*d*x + 1/2*c)^3 - 3913*tan(1/
2*d*x + 1/2*c)^2 + 3855*tan(1/2*d*x + 1/2*c) - 2275)/(5*tan(1/2*d*x + 1/2*
c)^2 - 6*tan(1/2*d*x + 1/2*c) + 5)^2 + 2950*arctan((3*cos(d*x + c) - sin(d
*x + c) + 3)/(cos(d*x + c) + 3*sin(d*x + c) - 9)))/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{1}{(5-3\sin(c+dx))^3}\,dx = \frac{59\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{3}{4}\right)}{1024\,d}-\frac{59\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d}+\frac{\frac{963\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1280}-\frac{11739\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400}+\frac{2313\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1280}-\frac{273}{256}}{d\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\right)^2}$$

input
```
int(-1/(3*sin(c + d*x) - 5)^3,x)
```

output
```
(59*atan((5*tan(c/2 + (d*x)/2))/4 - 3/4))/(1024*d) - (59*(atan(tan(c/2 + (
d*x)/2)) - (d*x)/2))/(1024*d) + ((2313*tan(c/2 + (d*x)/2))/1280 - (11739*t
an(c/2 + (d*x)/2)^2)/6400 + (963*tan(c/2 + (d*x)/2)^3)/1280 - 273/256)/(d*
(5*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) + 5)^2)
```

3.24. $\int \frac{1}{(5-3\sin(c+dx))^3}\,dx$

3.25 $\int \frac{1}{(5-3\sin(c+dx))^4}\,dx$

3.25.1 Optimal result

Integrand size = 12, antiderivative size = 108

$$\int \frac{1}{(5-3\sin(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{16384d} - \frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3} - \frac{25\cos(c+dx)}{512d(5-3\sin(c+dx))^2} - \frac{311\cos(c+dx)}{8192d(5-3\sin(c+dx))}$$

output
```
385/32768*x-385/16384*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d-1/16*cos(d*x+c)/
d/(5-3*sin(d*x+c))^3-25/512*cos(d*x+c)/d/(5-3*sin(d*x+c))^2-311/8192*cos(d
*x+c)/d/(5-3*sin(d*x+c))
```

3.25.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \frac{1}{(5-3\sin(c+dx))^4}\,dx = \frac{-1925\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{-239470+219735\cos(c+dx)+83970\cos(2(c+dx))-13995\cos(3(c+dx))+305091\sin}{2(-5+3\sin(c+dx))^3}}{81920d}$$

input
```
Integrate[(5 - 3*Sin[c + d*x])^(-4),x]
```

output
```
(-1925*ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])] + (-239470 + 219735*Cos[c + d*x] + 83970*Cos[2*(c + d
*x)] - 13995*Cos[3*(c + d*x)] + 305091*Sin[c + d*x] - 105300*Sin[2*(c + d*
x)] - 8397*Sin[3*(c + d*x)])/(2*(-5 + 3*Sin[c + d*x])^3))/(81920*d)
```

3.25.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3143, 27, 3042, 3233, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5-3\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5-3\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{48}\int -\frac{3(2\sin(c+dx)+5)}{(5-3\sin(c+dx))^3}dx-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{2\sin(c+dx)+5}{(5-3\sin(c+dx))^3}dx-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \frac{2\sin(c+dx)+5}{(5-3\sin(c+dx))^3}dx-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(-\frac{1}{32}\int -\frac{25\sin(c+dx)+62}{(5-3\sin(c+dx))^2}dx-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{25\sin(c+dx)+62}{(5-3\sin(c+dx))^2}dx-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{25\sin(c+dx)+62}{(5-3\sin(c+dx))^2}dx-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(-\frac{1}{16}\int -\frac{385}{5-3\sin(c+dx)}dx-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\left(\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}\right)-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

input
```
Int[(5 - 3*Sin[c + d*x])^(-4),x]
```

output
```
(((385*(x/4 - ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)))/16 - (311*Co
s[c + d*x])/(16*d*(5 - 3*Sin[c + d*x])))/32 - (25*Cos[c + d*x])/(32*d*(5 -
 3*Sin[c + d*x])^2))/16 - Cos[c + d*x]/(16*d*(5 - 3*Sin[c + d*x])^3)
```

3.25. $\int \frac{1}{(5-3\sin(c+dx))^4}\,dx$

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.25.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}-\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}-\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
default	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}-\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}-\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
risch	$-\frac{-239470\,e^{3i(dx+c)}-86625ie^{4i(dx+c)}+218466ie^{2i(dx+c)}+10395\,e^{5i(dx+c)}+73575\,e^{i(dx+c)}-8397i}{12288\left(3\,e^{2i(dx+c)}-3-10ie^{i(dx+c)}\right)^3 d}-\frac{385i\ln\left(e^{i(dx+c)}\right.}{32768d}$
parallelrisch	$31683960+48125i(770+27\sin(3dx+3c)-981\sin(dx+c)-270\cos(2dx+2c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)+48125i(-27\sin($

input
```
int(1/(5-3*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(250*(39933/5120000*tan(1/2*d*x+1/2*c)^5-672723/25600000*tan(1/2*d*x+1
/2*c)^4+2870073/64000000*tan(1/2*d*x+1/2*c)^3-604899/12800000*tan(1/2*d*x+
1/2*c)^2+145233/5120000*tan(1/2*d*x+1/2*c)-10287/1024000)/(5*tan(1/2*d*x+1
/2*c)^2-6*tan(1/2*d*x+1/2*c)+5)^3+385/16384*arctan(5/4*tan(1/2*d*x+1/2*c)-
3/4))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{1}{(5-3\sin(c+dx))^4}\,dx =$$
$$-\frac{11196\cos(dx+c)^3-385\left(135\cos(dx+c)^2-9\left(3\cos(dx+c)^2-28\right)\sin(dx+c)-260\right)\arctan\left(\frac{5\,\mathrm{s}}{4}\right.}{32768\left(135\,d\cos(dx+c)^2-9\left(3\,d\cos(dx+c)^2-28\,d\right)\mathrm{s}\right.}$$

input
```
integrate(1/(5-3*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/32768*(11196*cos(d*x + c)^3 - 385*(135*cos(d*x + c)^2 - 9*(3*cos(d*x +
c)^2 - 28)*sin(d*x + c) - 260)*arctan(1/4*(5*sin(d*x + c) - 3)/cos(d*x + c
)) + 42120*cos(d*x + c)*sin(d*x + c) - 52344*cos(d*x + c))/(135*d*cos(d*x
+ c)^2 - 9*(3*d*cos(d*x + c)^2 - 28*d)*sin(d*x + c) - 260*d)
```

3.25.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.39 (sec) , antiderivative size = 1690, normalized size of antiderivative = 15.65

$$\int \frac{1}{(5-3\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5-3*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(5 - 3*sin(2*atan(3/5 - 4*I/5)))**4, Eq(c, -d*x + 2*atan(3/5
 - 4*I/5))), (x/(5 - 3*sin(2*atan(3/5 + 4*I/5)))**4, Eq(c, -d*x + 2*atan(3/
5 + 4*I/5))), (x/(5 - 3*sin(c))**4, Eq(d, 0)), (6015625*(atan(5*tan(c/2 +
d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(
256000000*d*tan(c/2 + d*x/2)**6 - 921600000*d*tan(c/2 + d*x/2)**5 + 187392
0000*d*tan(c/2 + d*x/2)**4 - 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000
*d*tan(c/2 + d*x/2)**2 - 921600000*d*tan(c/2 + d*x/2) + 256000000*d) - 216
56250*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi
))*tan(c/2 + d*x/2)**5/(256000000*d*tan(c/2 + d*x/2)**6 - 921600000*d*tan(
c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)**4 - 2285568000*d*tan(c/2
+ d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2 - 921600000*d*tan(c/2 + d*x
/2) + 256000000*d) + 44034375*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor
((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(256000000*d*tan(c/2 + d*x/
2)**6 - 921600000*d*tan(c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)**4
 - 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2 - 9
21600000*d*tan(c/2 + d*x/2) + 256000000*d) - 53707500*(atan(5*tan(c/2 + d*
x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**3/(25
6000000*d*tan(c/2 + d*x/2)**6 - 921600000*d*tan(c/2 + d*x/2)**5 + 18739200
00*d*tan(c/2 + d*x/2)**4 - 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000*d
*tan(c/2 + d*x/2)**2 - 921600000*d*tan(c/2 + d*x/2) + 256000000*d) + 44...
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(97) = 194.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.34

$$\int \frac{1}{(5-3\sin(c+dx))^4}\,dx =$$

$$-\frac{\frac{36\left(\frac{403425\sin(dx+c)}{\cos(dx+c)+1}-\frac{672110\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{637794\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{373735\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{110925\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-142875\right)}{\frac{450\sin(dx+c)}{\cos(dx+c)+1}-\frac{915\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1116\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{915\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{450\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{125\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-125}-48125\arctan\left(\frac{5\sin}{4(\cos(}\right.}{2048000\,d}$$

input
```
integrate(1/(5-3*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/2048000*(36*(403425*sin(d*x + c)/(cos(d*x + c) + 1) - 672110*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 637794*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3
73735*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 110925*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 142875)/(450*sin(d*x + c)/(cos(d*x + c) + 1) - 915*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1116*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9
15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 450*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 - 125*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 125) - 48125*arctan(5/4
*sin(d*x + c)/(cos(d*x + c) + 1) - 3/4))/d
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5-3\sin(c+dx))^4}\,dx = \frac{48125\,dx+48125\,c+\frac{72\left(110925\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-373735\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+637794\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-672110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+403425}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^3}}{4096000\,d}$$

input
```
integrate(1/(5-3*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/4096000*(48125*d*x + 48125*c + 72*(110925*tan(1/2*d*x + 1/2*c)^5 - 37373
5*tan(1/2*d*x + 1/2*c)^4 + 637794*tan(1/2*d*x + 1/2*c)^3 - 672110*tan(1/2*
d*x + 1/2*c)^2 + 403425*tan(1/2*d*x + 1/2*c) - 142875)/(5*tan(1/2*d*x + 1/
2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 5)^3 + 96250*arctan((3*cos(d*x + c) - si
n(d*x + c) + 3)/(cos(d*x + c) + 3*sin(d*x + c) - 9)))/d
```

3.25.9 Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.73

$$\int \frac{1}{(5-3\sin(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{3}{4}\right)}{16384\,d} - \frac{385\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d} + \frac{\frac{39933\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2560000}-\frac{672723\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{12800000}+\frac{2870073\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000000}-\frac{604899\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400000}+\frac{145233\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2560000}-\frac{10287}{512000}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{25}-\frac{1116\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{125}+\frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{25}-\frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+1\right)}$$

input
```
int(1/(3*sin(c + d*x) - 5)^4,x)
```

output
```
(385*atan((5*tan(c/2 + (d*x)/2))/4 - 3/4))/(16384*d) - (385*(atan(tan(c/2
+ (d*x)/2)) - (d*x)/2))/(16384*d) + ((145233*tan(c/2 + (d*x)/2))/2560000 -
 (604899*tan(c/2 + (d*x)/2)^2)/6400000 + (2870073*tan(c/2 + (d*x)/2)^3)/32
000000 - (672723*tan(c/2 + (d*x)/2)^4)/12800000 + (39933*tan(c/2 + (d*x)/2
)^5)/2560000 - 10287/512000)/(d*((183*tan(c/2 + (d*x)/2)^2)/25 - (18*tan(c
/2 + (d*x)/2))/5 - (1116*tan(c/2 + (d*x)/2)^3)/125 + (183*tan(c/2 + (d*x)/
2)^4)/25 - (18*tan(c/2 + (d*x)/2)^5)/5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.25. $\int \frac{1}{(5-3\sin(c+dx))^4}\,dx$

3.26 $\int \frac{1}{-5+3\sin(c+dx)}\,dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{-5+3\sin(c+dx)}\,dx = -\frac{x}{4}+\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}$$

output
```
-1/4*x+1/2*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d
```

3.26.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{1}{-5+3\sin(c+dx)}\,dx = \frac{\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2d}$$

input
```
Integrate[(-5 + 3*Sin[c + d*x])^(-1),x]
```

output
```
ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])]/(2*d)
```

3.26.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3137}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sin(c+dx)-5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{3\sin(c+dx)-5}dx$$

$$\Big\downarrow \text{3137}$$

$$\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}-\frac{x}{4}$$

input
```
Int[(-5 + 3*Sin[c + d*x])^(-1),x]
```

output
```
-1/4*x + ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)
```

3.26.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3137
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

3.26.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{2d}$	20
default	$-\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{2d}$	20
risch	$\frac{i\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i}{3}\right)}{4d}-\frac{i\ln\left(\mathrm{e}^{i(dx+c)}-3i\right)}{4d}$	40
parallelrisch	$\frac{i\left(\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)-\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3+4i\right)\right)}{4d}$	42

input
```
int(1/(-5+3*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*arctan(5/4*tan(1/2*d*x+1/2*c)-3/4)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{-5+3\sin(c+dx)}\,dx = -\frac{\arctan\left(\frac{5\sin(dx+c)-3}{4\cos(dx+c)}\right)}{4\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*arctan(1/4*(5*sin(d*x + c) - 3)/cos(d*x + c))/d
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{1}{-5+3\sin(c+dx)}\,dx = \begin{cases} -\frac{\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0 \\ \frac{x}{3\sin(c)-5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-5+3*sin(d*x+c)),x)
```

output
```
Piecewise((-(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi
/2)/pi))/(2*d), Ne(d, 0)), (x/(3*sin(c) - 5), True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{-5+3\sin(c+dx)}\,dx = -\frac{\arctan\left(\frac{5\,\sin(dx+c)}{4\,(\cos(dx+c)+1)} - \frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) - 3/4)/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{1}{-5+3\sin(c+dx)}\,dx = -\frac{dx+c+2\,\arctan\left(\frac{3\,\cos(dx+c)-\sin(dx+c)+3}{\cos(dx+c)+3\,\sin(dx+c)-9}\right)}{4\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(d*x + c + 2*arctan((3*cos(d*x + c) - sin(d*x + c) + 3)/(cos(d*x + c)
 + 3*sin(d*x + c) - 9)))/d
```

3.26.9 Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{-5+3\sin(c+dx)}\,dx = \frac{\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}}{2\,d} - \frac{\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{3}{4}\right)}{2\,d}$$

input
```
int(1/(3*sin(c + d*x) - 5),x)
```

output
```
(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d) - atan((5*tan(c/2 + (d*x)/2))/4
 - 3/4)/(2*d)
```

3.27 $\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx$

3.27.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx=\frac{5x}{64}-\frac{5\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{32d}-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$

output
```
5/64*x-5/32*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d-3/16*cos(d*x+c)/d/(5-3*sin
(d*x+c))
```

3.27.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.57

$$\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx = \frac{-25\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)+\frac{6(-5+5\cos(c+dx)+3\sin(c+dx))}{-5+3\sin(c+dx)}}{160d}$$

input
```
Integrate[(-5 + 3*Sin[c + d*x])^(-2),x]
```

output
```
(-25*ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])] + (6*(-5 + 5*Cos[c + d*x] + 3*Sin[c + d*x]))/(-5 + 3*Si
n[c + d*x]))/(160*d)
```

3.27.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3143, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)-5)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(3\sin(c+dx)-5)^2}dx$$
$$\Big\downarrow \text{3143}$$
$$-\frac{1}{16}\int -\frac{5}{5-3\sin(c+dx)}dx-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$
$$\Big\downarrow \text{27}$$
$$\frac{5}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$
$$\Big\downarrow \text{3042}$$
$$\frac{5}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$
$$\Big\downarrow \text{3136}$$
$$\frac{5}{16}\left(\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}\right)-\frac{3\cos(c+dx)}{16d(5-3\sin(c+dx))}$$

input
```
Int[(-5 + 3*Sin[c + d*x])^(-2),x]
```

output
```
(5*(x/4 - ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)))/16 - (3*Cos[c + 
d*x])/(16*d*(5 - 3*Sin[c + d*x]))
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.27.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}-\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{32}}{d}$
default	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}-\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{32}}{d}$
risch	$-\frac{5\,\mathrm{e}^{i(dx+c)}-3i}{8d\left(3\,\mathrm{e}^{2i(dx+c)}-3-10i\mathrm{e}^{i(dx+c)}\right)}+\frac{5i\ln\left(\mathrm{e}^{i(dx+c)}-3i\right)}{64d}-\frac{5i\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i}{3}\right)}{64d}$
parallelrisch	$\frac{(-75i\sin(dx+c)+125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)+(75i\sin(dx+c)-125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3+4i\right)+60\cos(dx+c)-36}{960d\sin(dx+c)-1600d}$

input
```
int(1/(-5+3*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(9/400*tan(1/2*d*x+1/2*c)-3/80)/(tan(1/2*d*x+1/2*c)^2-6/5*tan(1/2*d
*x+1/2*c)+1)+5/32*arctan(5/4*tan(1/2*d*x+1/2*c)-3/4))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx = \frac{5\,(3\,\sin{(dx+c)}-5)\arctan\left(\frac{5\,\sin(dx+c)-3}{4\,\cos(dx+c)}\right)+12\,\cos{(dx+c)}}{64\,(3\,d\sin{(dx+c)}-5\,d)}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/64*(5*(3*sin(d*x + c) - 5)*arctan(1/4*(5*sin(d*x + c) - 3)/cos(d*x + c))
 + 12*cos(d*x + c))/(3*d*sin(d*x + c) - 5*d)
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.62

$$\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{x}{\left(-5+3\sin\left(2\,\mathrm{atan}\left(\frac{3}{5}-\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{\left(-5+3\sin\left(2\,\mathrm{atan}\left(\frac{3}{5}+\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{(3\sin{(c)}-5)^2} \\ \frac{125\left(\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} - \frac{150\left(\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} + \frac{125(}{800} \end{cases}$$

input
```
integrate(1/(-5+3*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(-5 + 3*sin(2*atan(3/5 - 4*I/5)))**2, Eq(c, -d*x + 2*atan(3/5
 - 4*I/5))), (x/(-5 + 3*sin(2*atan(3/5 + 4*I/5)))**2, Eq(c, -d*x + 2*atan(
3/5 + 4*I/5))), (x/(3*sin(c) - 5)**2, Eq(d, 0)), (125*(atan(5*tan(c/2 + d*
x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(80
0*d*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) - 150*(atan(5*ta
n(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x
/2)/(800*d*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) + 125*(at
an(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(800*d
*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) + 36*tan(c/2 + d*x/
2)/(800*d*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d) - 60/(800*
d*tan(c/2 + d*x/2)**2 - 960*d*tan(c/2 + d*x/2) + 800*d), True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

$$\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx = -\frac{\frac{12\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-5\right)}{\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-5} - 25\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}-\frac{3}{4}\right)}{160\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/160*(12*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5)/(6*sin(d*x + c)/(cos(d*
x + c) + 1) - 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5) - 25*arctan(5/4*s
in(d*x + c)/(cos(d*x + c) + 1) - 3/4))/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

$$\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx = \frac{25\,dx+25\,c+\frac{24\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\right)}{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}+50\arctan\left(\frac{3\cos(dx+c)-\sin(dx+c)+3}{\cos(dx+c)+3\sin(dx+c)-9}\right)}{320\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/320*(25*d*x + 25*c + 24*(3*tan(1/2*d*x + 1/2*c) - 5)/(5*tan(1/2*d*x + 1/
2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 5) + 50*arctan((3*cos(d*x + c) - sin(d*x
 + c) + 3)/(cos(d*x + c) + 3*sin(d*x + c) - 9)))/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

$$\int \frac{1}{(-5+3\sin(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{3}{4}\right)}{32\,d} - \frac{5\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{32\,d} + \frac{\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{200}-\frac{3}{40}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}+1\right)}$$

input
```
int(1/(3*sin(c + d*x) - 5)^2,x)
```

output
```
(5*atan((5*tan(c/2 + (d*x)/2))/4 - 3/4))/(32*d) - (5*(atan(tan(c/2 + (d*x)
/2)) - (d*x)/2))/(32*d) + ((9*tan(c/2 + (d*x)/2))/200 - 3/40)/(d*(tan(c/2
+ (d*x)/2)^2 - (6*tan(c/2 + (d*x)/2))/5 + 1))
```

3.28 $\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx = -\frac{59x}{2048}+\frac{59\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{1024d}+\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}+\frac{45\cos(c+dx)}{512d(5-3\sin(c+dx))}$$

output
```
-59/2048*x+59/1024*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d+3/32*cos(d*x+c)/d/(
5-3*sin(d*x+c))^2+45/512*cos(d*x+c)/d/(5-3*sin(d*x+c))
```

3.28.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx = \frac{59\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)+\frac{546\cos(c+dx)+9(-59+9\cos(2(c+dx))+60\sin(c+dx)-15\sin(2(c+dx)))}{(5-3\sin(c+dx))^2}}{1024d}$$

input
```
Integrate[(-5 + 3*Sin[c + d*x])^(-3),x]
```

output
```
(59*ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])] + (546*Cos[c + d*x] + 9*(-59 + 9*Cos[2*(c + d*x)] + 60*S
in[c + d*x] - 15*Sin[2*(c + d*x)]))/(5 - 3*Sin[c + d*x])^2)/(1024*d)
```

3.28.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3143, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)-5)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(c+dx)-5)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}-\frac{1}{32}\int \frac{3\sin(c+dx)+10}{(5-3\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}-\frac{1}{32}\int \frac{3\sin(c+dx)+10}{(5-3\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(\frac{1}{16}\int -\frac{59}{5-3\sin(c+dx)}dx+\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)+\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}-\frac{59}{16}\int \frac{1}{5-3\sin(c+dx)}dx\right)+\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}-\frac{59}{16}\int \frac{1}{5-3\sin(c+dx)}dx\right)+\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(5-3\sin(c+dx))}-\frac{59}{16}\left(\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}\right)\right)+\frac{3\cos(c+dx)}{32d(5-3\sin(c+dx))^2}$$

input
```
Int[(-5 + 3*Sin[c + d*x])^(-3),x]
```

output
```
((-59*(x/4 - ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)))/16 + (45*Cos[
c + d*x])/(16*d*(5 - 3*Sin[c + d*x])))/32 + (3*Cos[c + d*x])/(32*d*(5 - 3*
Sin[c + d*x])^2)
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.28.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{50\left(\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64000}-\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320000}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64000}-\frac{273}{12800}\right)}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}-\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{1024}}{d}$
default	$\frac{-\frac{50\left(\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64000}-\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320000}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64000}-\frac{273}{12800}\right)}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}-\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{3}{4}\right)}{1024}}{d}$
risch	$\frac{-\frac{885i e^{2i(dx+c)}}{256}+\frac{177 e^{3i(dx+c)}}{256}-\frac{723 e^{i(dx+c)}}{256}+\frac{135i}{256}}{\left(3 e^{2i(dx+c)}-3-10i e^{i(dx+c)}\right)^2 d}+\frac{59i\ln\left(e^{i(dx+c)}-\frac{i}{3}\right)}{2048d}-\frac{59i\ln\left(e^{i(dx+c)}-3i\right)}{2048d}$
parallelrisch	$\frac{-1062+59i(-59+9\cos(2dx+2c)+60\sin(dx+c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)+59i(59-9\cos(2dx+2c)-60\sin(dx+c))\ln\left(5\right.}{2048d(-59+9\cos(2dx+2c)+60\sin}$

input
```
int(1/(-5+3*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-50*(963/64000*tan(1/2*d*x+1/2*c)^3-11739/320000*tan(1/2*d*x+1/2*c)^2
+2313/64000*tan(1/2*d*x+1/2*c)-273/12800)/(5*tan(1/2*d*x+1/2*c)^2-6*tan(1/
2*d*x+1/2*c)+5)^2-59/1024*arctan(5/4*tan(1/2*d*x+1/2*c)-3/4))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx = -\frac{59\left(9\cos\left(dx+c\right)^2+30\sin\left(dx+c\right)-34\right)\arctan\left(\frac{5\sin(dx+c)-3}{4\cos(dx+c)}\right)-540\cos\left(dx+c\right)\sin\left(dx+c\right)+1092}{2048\left(9\,d\cos\left(dx+c\right)^2+30\,d\sin\left(dx+c\right)-34\,d\right)}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2048*(59*(9*cos(d*x + c)^2 + 30*sin(d*x + c) - 34)*arctan(1/4*(5*sin(d*
x + c) - 3)/cos(d*x + c)) - 540*cos(d*x + c)*sin(d*x + c) + 1092*cos(d*x +
 c))/(9*d*cos(d*x + c)^2 + 30*d*sin(d*x + c) - 34*d)
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 915, normalized size of antiderivative = 11.02

$$\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-5+3*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(-5 + 3*sin(2*atan(3/5 - 4*I/5)))**3, Eq(c, -d*x + 2*atan(3/5
 - 4*I/5))), (x/(-5 + 3*sin(2*atan(3/5 + 4*I/5)))**3, Eq(c, -d*x + 2*atan(
3/5 + 4*I/5))), (x/(3*sin(c) - 5)**3, Eq(d, 0)), (-36875*(atan(5*tan(c/2 +
 d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/
(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/2 + d*x/2)**3 + 2201600*d*
tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2) + 640000*d) + 88500*(atan
(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2
+ d*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/2 + d*x/2)**3
+ 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2) + 640000*d) -
 126850*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/
pi))*tan(c/2 + d*x/2)**2/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/2
 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2)
+ 640000*d) + 88500*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*
x/2 - pi/2)/pi))*tan(c/2 + d*x/2)/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*
d*tan(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2
+ d*x/2) + 640000*d) - 36875*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor(
(c/2 + d*x/2 - pi/2)/pi))/(640000*d*tan(c/2 + d*x/2)**4 - 1536000*d*tan(c/
2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*d*tan(c/2 + d*x/2)
 + 640000*d) - 19260*tan(c/2 + d*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 - 1
536000*d*tan(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 - 1536000*...
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.08

$$\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx = \frac{\frac{12\left(\frac{3855\sin(dx+c)}{\cos(dx+c)+1}-\frac{3913\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1605\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-2275\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1}-\frac{86\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-25}-1475\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}-\frac{3}{4}\right)}{25600\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/25600*(12*(3855*sin(d*x + c)/(cos(d*x + c) + 1) - 3913*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1605*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2275)/(60*
sin(d*x + c)/(cos(d*x + c) + 1) - 86*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 60*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 25) - 1475*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) - 3/4))/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx = -\frac{1475\,dx + 1475\,c + \frac{24\left(1605\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3913\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2275\right)}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^2} + 2950\,\arctan\left(\frac{3\,\cos(dx+}{\cos(dx+c)}\right.}{51200\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/51200*(1475*d*x + 1475*c + 24*(1605*tan(1/2*d*x + 1/2*c)^3 - 3913*tan(1
/2*d*x + 1/2*c)^2 + 3855*tan(1/2*d*x + 1/2*c) - 2275)/(5*tan(1/2*d*x + 1/2
*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 5)^2 + 2950*arctan((3*cos(d*x + c) - sin(
d*x + c) + 3)/(cos(d*x + c) + 3*sin(d*x + c) - 9)))/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx = \frac{59\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d} - \frac{59\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{3}{4}\right)}{1024\,d} - \frac{\frac{963\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1280}-\frac{11739\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400}+\frac{2313\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1280}-\frac{273}{256}}{d\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\right)^2}$$

input
```
int(1/(3*sin(c + d*x) - 5)^3,x)
```

output
```
(59*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(1024*d) - (59*atan((5*tan(c/2 +
 (d*x)/2))/4 - 3/4))/(1024*d) - ((2313*tan(c/2 + (d*x)/2))/1280 - (11739*t
an(c/2 + (d*x)/2)^2)/6400 + (963*tan(c/2 + (d*x)/2)^3)/1280 - 273/256)/(d*
(5*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) + 5)^2)
```

3.28. $\int \frac{1}{(-5+3\sin(c+dx))^3}\,dx$

3.29 $\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx$

3.29.1 Optimal result

Integrand size = 12, antiderivative size = 108

$$\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{16384d} - \frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3} - \frac{25\cos(c+dx)}{512d(5-3\sin(c+dx))^2} - \frac{311\cos(c+dx)}{8192d(5-3\sin(c+dx))}$$

output
```
385/32768*x-385/16384*arctan(cos(d*x+c)/(3-sin(d*x+c)))/d-1/16*cos(d*x+c)/
d/(5-3*sin(d*x+c))^3-25/512*cos(d*x+c)/d/(5-3*sin(d*x+c))^2-311/8192*cos(d
*x+c)/d/(5-3*sin(d*x+c))
```

3.29.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx = \frac{-1925\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{-239470+219735\cos(c+dx)+83970\cos(2(c+dx))-13995\cos(3(c+dx))+305091\sin}{2(-5+3\sin(c+dx))^3}}{81920d}$$

input
```
Integrate[(-5 + 3*Sin[c + d*x])^(-4),x]
```

output
```
(-1925*ArcTan[(2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])] + (-239470 + 219735*Cos[c + d*x] + 83970*Cos[2*(c + d
*x)] - 13995*Cos[3*(c + d*x)] + 305091*Sin[c + d*x] - 105300*Sin[2*(c + d*
x)] - 8397*Sin[3*(c + d*x)])/(2*(-5 + 3*Sin[c + d*x])^3))/(81920*d)
```

3.29.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3143, 27, 3042, 3233, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)-5)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(c+dx)-5)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{48}\int -\frac{3(2\sin(c+dx)+5)}{(5-3\sin(c+dx))^3}dx-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{2\sin(c+dx)+5}{(5-3\sin(c+dx))^3}dx-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \frac{2\sin(c+dx)+5}{(5-3\sin(c+dx))^3}dx-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(-\frac{1}{32}\int -\frac{25\sin(c+dx)+62}{(5-3\sin(c+dx))^2}dx-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{25\sin(c+dx)+62}{(5-3\sin(c+dx))^2}dx-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{25\sin(c+dx)+62}{(5-3\sin(c+dx))^2}dx-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$\big\downarrow$ 3233

$$\frac{1}{16}\left(\frac{1}{32}\left(-\frac{1}{16}\int -\frac{385}{5-3\sin(c+dx)}dx-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$\big\downarrow$ 27

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$\big\downarrow$ 3042

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{5-3\sin(c+dx)}dx-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

$\big\downarrow$ 3136

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\left(\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3-\sin(c+dx)}\right)}{2d}\right)-\frac{311\cos(c+dx)}{16d(5-3\sin(c+dx))}\right)-\frac{25\cos(c+dx)}{32d(5-3\sin(c+dx))^2}\right)-\frac{\cos(c+dx)}{16d(5-3\sin(c+dx))^3}$$

input
```
Int[(-5 + 3*Sin[c + d*x])^(-4),x]
```

output
```
(((385*(x/4 - ArcTan[Cos[c + d*x]/(3 - Sin[c + d*x])]/(2*d)))/16 - (311*Co
s[c + d*x])/(16*d*(5 - 3*Sin[c + d*x])))/32 - (25*Cos[c + d*x])/(32*d*(5 -
 3*Sin[c + d*x])^2))/16 - Cos[c + d*x]/(16*d*(5 - 3*Sin[c + d*x])^3)
```

3.29. $\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.29.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}-\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}-\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
default	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}-\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}-\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
risch	$-\frac{-239470\,e^{3i(dx+c)}-86625ie^{4i(dx+c)}+218466ie^{2i(dx+c)}+10395\,e^{5i(dx+c)}+73575\,e^{i(dx+c)}-8397i}{12288\left(3\,e^{2i(dx+c)}-3-10ie^{i(dx+c)}\right)^3 d}-\frac{385i\ln\left(e^{i(dx+c)}\right.}{32768d}$
parallelrisch	$\frac{31683960+48125i(770+27\sin(3dx+3c)-981\sin(dx+c)-270\cos(2dx+2c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3-4i\right)+48125i(-27\sin(}{}$

input
```
int(1/(-5+3*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(250*(39933/5120000*tan(1/2*d*x+1/2*c)^5-672723/25600000*tan(1/2*d*x+1
/2*c)^4+2870073/64000000*tan(1/2*d*x+1/2*c)^3-604899/12800000*tan(1/2*d*x+
1/2*c)^2+145233/5120000*tan(1/2*d*x+1/2*c)-10287/1024000)/(5*tan(1/2*d*x+1
/2*c)^2-6*tan(1/2*d*x+1/2*c)+5)^3+385/16384*arctan(5/4*tan(1/2*d*x+1/2*c)-
3/4))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx =$$
$$-\frac{11196\,\cos(dx+c)^3-385\left(135\,\cos(dx+c)^2-9\left(3\,\cos(dx+c)^2-28\right)\sin(dx+c)-260\right)\arctan\left(\frac{5\,\mathrm{s}}{4}\right.}{32768\left(135\,d\cos(dx+c)^2-9\left(3\,d\cos(dx+c)^2-28\,d\right)\mathrm{s}\right.}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/32768*(11196*cos(d*x + c)^3 - 385*(135*cos(d*x + c)^2 - 9*(3*cos(d*x +
c)^2 - 28)*sin(d*x + c) - 260)*arctan(1/4*(5*sin(d*x + c) - 3)/cos(d*x + c
)) + 42120*cos(d*x + c)*sin(d*x + c) - 52344*cos(d*x + c))/(135*d*cos(d*x
+ c)^2 - 9*(3*d*cos(d*x + c)^2 - 28*d)*sin(d*x + c) - 260*d)
```

3.29.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.32 (sec) , antiderivative size = 1690, normalized size of antiderivative = 15.65

$$\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-5+3*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(-5 + 3*sin(2*atan(3/5 - 4*I/5)))**4, Eq(c, -d*x + 2*atan(3/5
 - 4*I/5))), (x/(-5 + 3*sin(2*atan(3/5 + 4*I/5)))**4, Eq(c, -d*x + 2*atan(
3/5 + 4*I/5))), (x/(3*sin(c) - 5)**4, Eq(d, 0)), (6015625*(atan(5*tan(c/2
+ d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6
/(256000000*d*tan(c/2 + d*x/2)**6 - 921600000*d*tan(c/2 + d*x/2)**5 + 1873
920000*d*tan(c/2 + d*x/2)**4 - 2285568000*d*tan(c/2 + d*x/2)**3 + 18739200
00*d*tan(c/2 + d*x/2)**2 - 921600000*d*tan(c/2 + d*x/2) + 256000000*d) - 2
1656250*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/
pi))*tan(c/2 + d*x/2)**5/(256000000*d*tan(c/2 + d*x/2)**6 - 921600000*d*ta
n(c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)**4 - 2285568000*d*tan(c/
2 + d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2 - 921600000*d*tan(c/2 + d
*x/2) + 256000000*d) + 44034375*(atan(5*tan(c/2 + d*x/2)/4 - 3/4) + pi*flo
or((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(256000000*d*tan(c/2 + d*
x/2)**6 - 921600000*d*tan(c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)*
*4 - 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2 -
 921600000*d*tan(c/2 + d*x/2) + 256000000*d) - 53707500*(atan(5*tan(c/2 +
d*x/2)/4 - 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**3/(
256000000*d*tan(c/2 + d*x/2)**6 - 921600000*d*tan(c/2 + d*x/2)**5 + 187392
0000*d*tan(c/2 + d*x/2)**4 - 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000
*d*tan(c/2 + d*x/2)**2 - 921600000*d*tan(c/2 + d*x/2) + 256000000*d) + ...
```

3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(97) = 194.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.34

$$\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx =$$

$$-\frac{\frac{36\left(\frac{403425\sin(dx+c)}{\cos(dx+c)+1}-\frac{672110\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{637794\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{373735\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{110925\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-142875\right)}{\frac{450\sin(dx+c)}{\cos(dx+c)+1}-\frac{915\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1116\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{915\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{450\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{125\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-125}-48125\arctan\left(\frac{5\sin}{4(\cos(}\right.}{2048000\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/2048000*(36*(403425*sin(d*x + c)/(cos(d*x + c) + 1) - 672110*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 637794*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3
73735*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 110925*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 142875)/(450*sin(d*x + c)/(cos(d*x + c) + 1) - 915*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1116*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9
15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 450*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 - 125*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 125) - 48125*arctan(5/4
*sin(d*x + c)/(cos(d*x + c) + 1) - 3/4))/d
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.37

$$\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx$$
$$= \frac{48125\,dx + 48125\,c + \frac{72\left(110925\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 373735\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 637794\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 672110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 403425 \cdots}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^3}}{4096000\,d}$$

input
```
integrate(1/(-5+3*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/4096000*(48125*d*x + 48125*c + 72*(110925*tan(1/2*d*x + 1/2*c)^5 - 37373
5*tan(1/2*d*x + 1/2*c)^4 + 637794*tan(1/2*d*x + 1/2*c)^3 - 672110*tan(1/2*
d*x + 1/2*c)^2 + 403425*tan(1/2*d*x + 1/2*c) - 142875)/(5*tan(1/2*d*x + 1/
2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 5)^3 + 96250*arctan((3*cos(d*x + c) - si
n(d*x + c) + 3)/(cos(d*x + c) + 3*sin(d*x + c) - 9)))/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.73

$$\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{3}{4}\right)}{16384\,d} - \frac{385\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d}$$
$$+ \frac{\frac{39933\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2560000} - \frac{672723\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{12800000} + \frac{2870073\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000000} - \frac{604899\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400000} + \frac{145233\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2560000} - \frac{10287}{512000} \cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - \frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5} + \frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{25} - \frac{1116\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{125} + \frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{25} - \frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5} + 1 \cdots\right.}$$

input
```
int(1/(3*sin(c + d*x) - 5)^4,x)
```

output
```
(385*atan((5*tan(c/2 + (d*x)/2))/4 - 3/4))/(16384*d) - (385*(atan(tan(c/2
+ (d*x)/2)) - (d*x)/2))/(16384*d) + ((145233*tan(c/2 + (d*x)/2))/2560000 -
 (604899*tan(c/2 + (d*x)/2)^2)/6400000 + (2870073*tan(c/2 + (d*x)/2)^3)/32
000000 - (672723*tan(c/2 + (d*x)/2)^4)/12800000 + (39933*tan(c/2 + (d*x)/2
)^5)/2560000 - 10287/512000)/(d*((183*tan(c/2 + (d*x)/2)^2)/25 - (18*tan(c
/2 + (d*x)/2))/5 - (1116*tan(c/2 + (d*x)/2)^3)/125 + (183*tan(c/2 + (d*x)/
2)^4)/25 - (18*tan(c/2 + (d*x)/2)^5)/5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.29. $\int \frac{1}{(-5+3\sin(c+dx))^4}\,dx$

3.30 $\int \frac{1}{-5-3\sin(c+dx)}\,dx$

3.30.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{-5-3\sin(c+dx)}\,dx = -\frac{x}{4}-\frac{\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{2d}$$

output
```
-1/4*x-1/2*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{-5-3\sin(c+dx)}\,dx = -\frac{\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2d}$$

input
```
Integrate[(-5 - 3*Sin[c + d*x])^(-1),x]
```

output
```
-1/2*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])]/d
```

3.30.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3137}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3\sin(c+dx)-5}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{-3\sin(c+dx)-5}dx$$

$$\Big\downarrow \text{ 3137}$$

$$-\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}-\frac{x}{4}$$

input
```
Int[(-5 - 3*Sin[c + d*x])^(-1),x]
```

output
```
-1/4*x - ArcTan[Cos[c + d*x]/(3 + Sin[c + d*x])]/(2*d)
```

3.30.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3137
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

3.30.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{2d}$	20
default	$-\frac{\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{2d}$	20
risch	$-\frac{i\ln\left(\mathrm{e}^{i(dx+c)}+3i\right)}{4d}+\frac{i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i}{3}\right)}{4d}$	40
parallelrisch	$\frac{i\left(\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)-\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3+4i\right)\right)}{4d}$	42

input
```
int(1/(-5-3*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*arctan(5/4*tan(1/2*d*x+1/2*c)+3/4)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{-5-3\sin(c+dx)}\,dx = -\frac{\arctan\left(\frac{5\sin(dx+c)+3}{4\cos(dx+c)}\right)}{4d}$$

input
```
integrate(1/(-5-3*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c))/d
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{1}{-5-3\sin(c+dx)}\,dx = \begin{cases} -\frac{\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0 \\ \frac{x}{-3\sin(c)-5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-5-3*sin(d*x+c)),x)
```

output
```
Piecewise((-(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi
/2)/pi))/(2*d), Ne(d, 0)), (x/(-3*sin(c) - 5), True))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{-5-3\sin(c+dx)}\,dx = -\frac{\arctan\left(\frac{5\,\sin(dx+c)}{4\,(\cos(dx+c)+1)}+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) + 3/4)/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{1}{-5-3\sin(c+dx)}\,dx = -\frac{dx+c+2\,\arctan\left(-\frac{3\,\cos(dx+c)+\sin(dx+c)+3}{\cos(dx+c)-3\,\sin(dx+c)-9}\right)}{4\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(d*x + c + 2*arctan(-(3*cos(d*x + c) + sin(d*x + c) + 3)/(cos(d*x + c
) - 3*sin(d*x + c) - 9)))/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{-5-3\sin(c+dx)}\,dx = \frac{\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}}{2\,d} - \frac{\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3}{4}\right)}{2\,d}$$

input
```
int(-1/(3*sin(c + d*x) + 5),x)
```

output
```
(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d) - atan((5*tan(c/2 + (d*x)/2))/4
 + 3/4)/(2*d)
```

3.31 $\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx$

3.31.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx = \frac{5x}{64} + \frac{5\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{32d} + \frac{3\cos(c+dx)}{16d(5+3\sin(c+dx))}$$

output
```
5/64*x+5/32*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d+3/16*cos(d*x+c)/d/(5+3*sin
(d*x+c))
```

3.31.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx = \frac{25\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{6(-5+5\cos(c+dx)-3\sin(c+dx))}{5+3\sin(c+dx)}}{160d}$$

input
```
Integrate[(-5 - 3*Sin[c + d*x])^(-2),x]
```

output
```
(25*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])] + (6*(-5 + 5*Cos[c + d*x] - 3*Sin[c + d*x]))/(5 + 3*Sin[
c + d*x]))/(160*d)
```

3.31.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3143, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3\sin(c+dx)-5)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(-3\sin(c+dx)-5)^2}dx$$
$$\Big\downarrow \text{ 3143}$$
$$\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}-\frac{1}{16}\int -\frac{5}{3\sin(c+dx)+5}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{5}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{5}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}$$
$$\Big\downarrow \text{ 3136}$$
$$\frac{5}{16}\left(\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}+\frac{x}{4}\right)+\frac{3\cos(c+dx)}{16d(3\sin(c+dx)+5)}$$

input
```
Int[(-5 - 3*Sin[c + d*x])^(-2),x]
```

output
```
(5*(x/4 + ArcTan[Cos[c + d*x]/(3 + Sin[c + d*x])]/(2*d)))/16 + (3*Cos[c +
d*x])/(16*d*(5 + 3*Sin[c + d*x]))
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.31.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}+\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{32}}{d}$
default	$\frac{\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{200}+\frac{3}{40}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+1}+\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{32}}{d}$
risch	$\frac{5\,e^{i(dx+c)}+3i}{8d\left(3\,e^{2i(dx+c)}-3+10i e^{i(dx+c)}\right)}+\frac{5i\ln\left(e^{i(dx+c)}+3i\right)}{64d}-\frac{5i\ln\left(e^{i(dx+c)}+\frac{i}{3}\right)}{64d}$
parallelrisch	$\frac{(-75i\sin(dx+c)-125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)+(75i\sin(dx+c)+125i)\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3+4i\right)+60\cos(dx+c)+36}{960d\sin(dx+c)+1600d}$

input
```
int(1/(-5-3*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(9/400*tan(1/2*d*x+1/2*c)+3/80)/(tan(1/2*d*x+1/2*c)^2+6/5*tan(1/2*d
*x+1/2*c)+1)+5/32*arctan(5/4*tan(1/2*d*x+1/2*c)+3/4))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx = \frac{5\,(3\,\sin(dx+c)+5)\arctan\left(\frac{5\,\sin(dx+c)+3}{4\,\cos(dx+c)}\right)+12\,\cos(dx+c)}{64\,(3\,d\sin(dx+c)+5\,d)}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/64*(5*(3*sin(d*x + c) + 5)*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c))
 + 12*cos(d*x + c))/(3*d*sin(d*x + c) + 5*d)
```

3.31.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 389, normalized size of antiderivative = 6.95

$$\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{x}{\left(-5+3\sin\left(2\operatorname{atan}\left(\frac{3}{5}-\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{\left(-5+3\sin\left(2\operatorname{atan}\left(\frac{3}{5}+\frac{4i}{5}\right)\right)\right)^2} \\ \frac{x}{(-3\sin(c)-5)^2} \\ \frac{125\left(\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} + \frac{150\left(\operatorname{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{3}{4}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{800d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+960d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+800d} + \frac{125(}{800} \end{cases}$$

input
```
integrate(1/(-5-3*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(-5 + 3*sin(2*atan(3/5 - 4*I/5)))**2, Eq(c, -d*x - 2*atan(3/5
 - 4*I/5))), (x/(-5 + 3*sin(2*atan(3/5 + 4*I/5)))**2, Eq(c, -d*x - 2*atan(
3/5 + 4*I/5))), (x/(-3*sin(c) - 5)**2, Eq(d, 0)), (125*(atan(5*tan(c/2 + d
*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(8
00*d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 150*(atan(5*t
an(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*
x/2)/(800*d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 125*(a
tan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(800*
d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 36*tan(c/2 + d*x
/2)/(800*d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d) + 60/(800
*d*tan(c/2 + d*x/2)**2 + 960*d*tan(c/2 + d*x/2) + 800*d), True))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx = \frac{\frac{12\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+5\right)}{\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5}+25\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}+\frac{3}{4}\right)}{160\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/160*(12*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5)/(6*sin(d*x + c)/(cos(d*x
 + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5) + 25*arctan(5/4*si
n(d*x + c)/(cos(d*x + c) + 1) + 3/4))/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx = \frac{25\,dx+25\,c+\frac{24\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}+50\arctan\left(-\frac{3\cos(dx+c)+\sin(dx+c)+3}{\cos(dx+c)-3\sin(dx+c)-9}\right)}{320\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/320*(25*d*x + 25*c + 24*(3*tan(1/2*d*x + 1/2*c) + 5)/(5*tan(1/2*d*x + 1/
2*c)^2 + 6*tan(1/2*d*x + 1/2*c) + 5) + 50*arctan(-(3*cos(d*x + c) + sin(d*
x + c) + 3)/(cos(d*x + c) - 3*sin(d*x + c) - 9)))/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.48

$$\int \frac{1}{(-5-3\sin(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3}{4}\right)}{32\,d} - \frac{5\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{32\,d} + \frac{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{200}+\frac{3}{40}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+1\right)}$$

input
```
int(1/(3*sin(c + d*x) + 5)^2,x)
```

output
```
(5*atan((5*tan(c/2 + (d*x)/2))/4 + 3/4))/(32*d) - (5*(atan(tan(c/2 + (d*x)
/2)) - (d*x)/2))/(32*d) + ((9*tan(c/2 + (d*x)/2))/200 + 3/40)/(d*((6*tan(c
/2 + (d*x)/2))/5 + tan(c/2 + (d*x)/2)^2 + 1))
```

3.32 $\int \frac{1}{(-5-3\sin(c+dx))^3} dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(-5-3\sin(c+dx))^3} dx = -\frac{59x}{2048} - \frac{59\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{1024d} - \frac{3\cos(c+dx)}{32d(5+3\sin(c+dx))^2} - \frac{45\cos(c+dx)}{512d(5+3\sin(c+dx))}$$

output
```
-59/2048*x-59/1024*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d-3/32*cos(d*x+c)/d/(
5+3*sin(d*x+c))^2-45/512*cos(d*x+c)/d/(5+3*sin(d*x+c))
```

3.32.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{1}{(-5-3\sin(c+dx))^3} dx = \frac{-59\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{3(-182\cos(c+dx)+3(59-9\cos(2(c+dx))+60\sin(c+dx)-15\sin(2(c+dx))))}{(5+3\sin(c+dx))^2}}{1024d}$$

input
```
Integrate[(-5 - 3*Sin[c + d*x])^(-3),x]
```

output
```
(-59*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])] + (3*(-182*Cos[c + d*x] + 3*(59 - 9*Cos[2*(c + d*x)] +
60*Sin[c + d*x] - 15*Sin[2*(c + d*x)])))/(5 + 3*Sin[c + d*x])^2)/(1024*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3143, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3\sin(c+dx)-5)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(-3\sin(c+dx)-5)^3}dx$$

$$\Big\downarrow \text{ 3143}$$

$$-\frac{1}{32}\int \frac{10-3\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx-\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{32}\int \frac{10-3\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx-\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3233}$$

$$\frac{1}{32}\left(\frac{1}{16}\int -\frac{59}{3\sin(c+dx)+5}dx-\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)-\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{32}\left(-\frac{59}{16}\int \frac{1}{3\sin(c+dx)+5}dx-\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)-\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{32}\left(-\frac{59}{16}\int \frac{1}{3\sin(c+dx)+5}dx-\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)-\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

$$\Big\downarrow \text{ 3136}$$

$$\frac{1}{32}\left(-\frac{59}{16}\left(\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}+\frac{x}{4}\right)-\frac{45\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)-\frac{3\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}$$

input
```
Int[(-5 - 3*Sin[c + d*x])^(-3),x]
```

output
```
(-3*Cos[c + d*x])/(32*d*(5 + 3*Sin[c + d*x])^2) + ((-59*(x/4 + ArcTan[Cos[
c + d*x]/(3 + Sin[c + d*x])]/(2*d)))/16 - (45*Cos[c + d*x])/(16*d*(5 + 3*S
in[c + d*x])))/32
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.32.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-\frac{50\left(\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64000}+\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320000}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64000}+\frac{273}{12800}\right)}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}-\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{1024}}{d}$
default	$\frac{-\frac{50\left(\frac{963\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64000}+\frac{11739\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320000}+\frac{2313\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64000}+\frac{273}{12800}\right)}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^2}-\frac{59\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{1024}}{d}$
risch	$-\frac{3\left(295i\,e^{2i(dx+c)}+59\,e^{3i(dx+c)}-241\,e^{i(dx+c)}-45i\right)}{256\left(3\,e^{2i(dx+c)}-3+10i\,e^{i(dx+c)}\right)^2 d}+\frac{59i\ln\left(e^{i(dx+c)}+\frac{i}{3}\right)}{2048d}-\frac{59i\ln\left(e^{i(dx+c)}+3i\right)}{2048d}$
parallelrisch	$\frac{1062+59i(-59+9\cos(2dx+2c)-60\sin(dx+c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)+59i(59-9\cos(2dx+2c)+60\sin(dx+c))\ln\left(5\,\mathrm{t}\right.}{2048d(-59+9\cos(2dx+2c)-60\sin(}$

input
```
int(1/(-5-3*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-50*(963/64000*tan(1/2*d*x+1/2*c)^3+11739/320000*tan(1/2*d*x+1/2*c)^2
+2313/64000*tan(1/2*d*x+1/2*c)+273/12800)/(5*tan(1/2*d*x+1/2*c)^2+6*tan(1/
2*d*x+1/2*c)+5)^2-59/1024*arctan(5/4*tan(1/2*d*x+1/2*c)+3/4))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-5-3\sin(c+dx))^3}\,dx = -\frac{59\left(9\cos\left(dx+c\right)^2-30\sin\left(dx+c\right)-34\right)\arctan\left(\frac{5\sin(dx+c)+3}{4\cos(dx+c)}\right)-540\cos\left(dx+c\right)\sin\left(dx+c\right)-109}{2048\left(9\,d\cos\left(dx+c\right)^2-30\,d\sin\left(dx+c\right)-34\,d\right)}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2048*(59*(9*cos(d*x + c)^2 - 30*sin(d*x + c) - 34)*arctan(1/4*(5*sin(d*
x + c) + 3)/cos(d*x + c)) - 540*cos(d*x + c)*sin(d*x + c) - 1092*cos(d*x +
 c))/(9*d*cos(d*x + c)^2 - 30*d*sin(d*x + c) - 34*d)
```

3.32.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 921, normalized size of antiderivative = 11.37

$$\int \frac{1}{(-5-3\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-5-3*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(-5 + 3*sin(2*atan(3/5 - 4*I/5)))**3, Eq(c, -d*x - 2*atan(3/5
 - 4*I/5))), (x/(-5 + 3*sin(2*atan(3/5 + 4*I/5)))**3, Eq(c, -d*x - 2*atan(
3/5 + 4*I/5))), (x/(-3*sin(c) - 5)**3, Eq(d, 0)), (-36875*(atan(5*tan(c/2
+ d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4
/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c/2 + d*x/2)**3 + 2201600*d
*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2) + 640000*d) - 88500*(ata
n(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2
 + d*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c/2 + d*x/2)**3
 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2) + 640000*d)
- 126850*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)
/pi))*tan(c/2 + d*x/2)**2/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c/
2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2)
 + 640000*d) - 88500*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d
*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)/(640000*d*tan(c/2 + d*x/2)**4 + 1536000
*d*tan(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2
 + d*x/2) + 640000*d) - 36875*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor
((c/2 + d*x/2 - pi/2)/pi))/(640000*d*tan(c/2 + d*x/2)**4 + 1536000*d*tan(c
/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000*d*tan(c/2 + d*x/2
) + 640000*d) - 19260*tan(c/2 + d*x/2)**3/(640000*d*tan(c/2 + d*x/2)**4 +
1536000*d*tan(c/2 + d*x/2)**3 + 2201600*d*tan(c/2 + d*x/2)**2 + 1536000...
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(73) = 146$.

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.14

$$\int \frac{1}{(-5-3\sin(c+dx))^3}\,dx = -\frac{\frac{12\left(\frac{3855\sin(dx+c)}{\cos(dx+c)+1}+\frac{3913\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1605\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+2275\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1}+\frac{86\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+25}+1475\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}+\frac{3}{4}\right)}{25600\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/25600*(12*(3855*sin(d*x + c)/(cos(d*x + c) + 1) + 3913*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 1605*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2275)/(60
*sin(d*x + c)/(cos(d*x + c) + 1) + 86*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 60*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 25*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 25) + 1475*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) + 3/4))/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.49

$$\int \frac{1}{(-5-3\sin(c+dx))^3}\,dx = -\frac{1475\,dx + 1475\,c + \frac{24\left(1605\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3913\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2275\right)}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^2} + 2950\,\arctan\left(-\frac{3\,\cos(d}{\cos(dx+}\right.}{51200\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/51200*(1475*d*x + 1475*c + 24*(1605*tan(1/2*d*x + 1/2*c)^3 + 3913*tan(1
/2*d*x + 1/2*c)^2 + 3855*tan(1/2*d*x + 1/2*c) + 2275)/(5*tan(1/2*d*x + 1/2
*c)^2 + 6*tan(1/2*d*x + 1/2*c) + 5)^2 + 2950*arctan(-(3*cos(d*x + c) + sin
(d*x + c) + 3)/(cos(d*x + c) - 3*sin(d*x + c) - 9)))/d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{1}{(-5-3\sin(c+dx))^3}\,dx = \frac{59\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d} - \frac{59\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3}{4}\right)}{1024\,d} - \frac{\frac{963\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1280}+\frac{11739\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400}+\frac{2313\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1280}+\frac{273}{256}}{d\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\right)^2}$$

input
```
int(-1/(3*sin(c + d*x) + 5)^3,x)
```

output
```
(59*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(1024*d) - (59*atan((5*tan(c/2 +
 (d*x)/2))/4 + 3/4))/(1024*d) - ((2313*tan(c/2 + (d*x)/2))/1280 + (11739*t
an(c/2 + (d*x)/2)^2)/6400 + (963*tan(c/2 + (d*x)/2)^3)/1280 + 273/256)/(d*
(6*tan(c/2 + (d*x)/2) + 5*tan(c/2 + (d*x)/2)^2 + 5)^2)
```

3.32. $\int \frac{1}{(-5-3\sin(c+dx))^3}\,dx$

3.33 $\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx = \frac{385x}{32768} + \frac{385\arctan\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{16384d} + \frac{\cos(c+dx)}{16d(5+3\sin(c+dx))^3} + \frac{25\cos(c+dx)}{512d(5+3\sin(c+dx))^2} + \frac{311\cos(c+dx)}{8192d(5+3\sin(c+dx))}$$

output
```
385/32768*x+385/16384*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d+1/16*cos(d*x+c)/
d/(5+3*sin(d*x+c))^3+25/512*cos(d*x+c)/d/(5+3*sin(d*x+c))^2+311/8192*cos(d
*x+c)/d/(5+3*sin(d*x+c))
```

3.33.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx = \frac{1925\arctan\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right) + \frac{-239470+219735\cos(c+dx)+83970\cos(2(c+dx))-13995\cos(3(c+dx))-305091\sin(c}{2(5+3\sin(c+dx))^3}}{81920d}$$

input
```
Integrate[(-5 - 3*Sin[c + d*x])^(-4),x]
```

output
```
(1925*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])] + (-239470 + 219735*Cos[c + d*x] + 83970*Cos[2*(c + d*
x)] - 13995*Cos[3*(c + d*x)] - 305091*Sin[c + d*x] + 105300*Sin[2*(c + d*x
)] + 8397*Sin[3*(c + d*x)])/(2*(5 + 3*Sin[c + d*x])^3))/(81920*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3143, 27, 3042, 3233, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3\sin(c+dx)-5)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-3\sin(c+dx)-5)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}-\frac{1}{48}\int -\frac{3(5-2\sin(c+dx))}{(3\sin(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{5-2\sin(c+dx)}{(3\sin(c+dx)+5)^3}dx+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \frac{5-2\sin(c+dx)}{(3\sin(c+dx)+5)^3}dx+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}-\frac{1}{32}\int -\frac{62-25\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25\sin(c+dx)}{(3\sin(c+dx)+5)^2}dx+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$\Big\downarrow$ 3233

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}-\frac{1}{16}\int -\frac{385}{3\sin(c+dx)+5}dx\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$\Big\downarrow$ 27

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$\Big\downarrow$ 3042

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3\sin(c+dx)+5}dx+\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

$\Big\downarrow$ 3136

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\left(\frac{\arctan\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{2d}+\frac{x}{4}\right)+\frac{311\cos(c+dx)}{16d(3\sin(c+dx)+5)}\right)+\frac{25\cos(c+dx)}{32d(3\sin(c+dx)+5)^2}\right)+\frac{\cos(c+dx)}{16d(3\sin(c+dx)+5)^3}$$

input
```
Int[(-5 - 3*Sin[c + d*x])^(-4),x]
```

output
```
Cos[c + d*x]/(16*d*(5 + 3*Sin[c + d*x])^3) + ((25*Cos[c + d*x])/(32*d*(5 +
 3*Sin[c + d*x])^2) + ((385*(x/4 + ArcTan[Cos[c + d*x]/(3 + Sin[c + d*x])]
/(2*d)))/16 + (311*Cos[c + d*x])/(16*d*(5 + 3*Sin[c + d*x])))/32)/16
```

3.33. $\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx$

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.33.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}+\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}+\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}+\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
default	$\frac{\frac{39933\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}+\frac{672723\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{102400}+\frac{2870073\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256000}+\frac{604899\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{51200}+\frac{145233\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}+\frac{10287}{4096}}{\left(5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\right)^3}}{d}$
risch	$\frac{-239470\,e^{3i(dx+c)}+86625ie^{4i(dx+c)}-218466ie^{2i(dx+c)}+10395\,e^{5i(dx+c)}+73575\,e^{i(dx+c)}+8397i}{12288\left(3\,e^{2i(dx+c)}-3+10ie^{i(dx+c)}\right)^3 d}-\frac{385i\ln\left(e^{i(dx+c)}+\right.}{32768d}$
parallelrisch	$\frac{-31683960+48125i(770-27\sin(3dx+3c)+981\sin(dx+c)-270\cos(2dx+2c))\ln\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3-4i\right)+48125i(27\sin(}{}$

input
```
int(1/(-5-3*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(250*(39933/5120000*tan(1/2*d*x+1/2*c)^5+672723/25600000*tan(1/2*d*x+1
/2*c)^4+2870073/64000000*tan(1/2*d*x+1/2*c)^3+604899/12800000*tan(1/2*d*x+
1/2*c)^2+145233/5120000*tan(1/2*d*x+1/2*c)+10287/1024000)/(5*tan(1/2*d*x+1
/2*c)^2+6*tan(1/2*d*x+1/2*c)+5)^3+385/16384*arctan(5/4*tan(1/2*d*x+1/2*c)+
3/4))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx$$
$$=\frac{11196\cos(dx+c)^3+385\left(135\cos(dx+c)^2+9\left(3\cos(dx+c)^2-28\right)\sin(dx+c)-260\right)\arctan\left(\frac{5\sin(}{4\cos}\right.}{32768\left(135\,d\cos(dx+c)^2+9\left(3\,d\cos(dx+c)^2-28\,d\right)\sin\right.}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/32768*(11196*cos(d*x + c)^3 + 385*(135*cos(d*x + c)^2 + 9*(3*cos(d*x + c
)^2 - 28)*sin(d*x + c) - 260)*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c)
) - 42120*cos(d*x + c)*sin(d*x + c) - 52344*cos(d*x + c))/(135*d*cos(d*x +
 c)^2 + 9*(3*d*cos(d*x + c)^2 - 28*d)*sin(d*x + c) - 260*d)
```

3.33.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.40 (sec) , antiderivative size = 1695, normalized size of antiderivative = 15.99

$$\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-5-3*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(-5 + 3*sin(2*atan(3/5 - 4*I/5)))**4, Eq(c, -d*x - 2*atan(3/5
 - 4*I/5))), (x/(-5 + 3*sin(2*atan(3/5 + 4*I/5)))**4, Eq(c, -d*x - 2*atan(
3/5 + 4*I/5))), (x/(-3*sin(c) - 5)**4, Eq(d, 0)), (6015625*(atan(5*tan(c/2
 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**
6/(256000000*d*tan(c/2 + d*x/2)**6 + 921600000*d*tan(c/2 + d*x/2)**5 + 187
3920000*d*tan(c/2 + d*x/2)**4 + 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920
000*d*tan(c/2 + d*x/2)**2 + 921600000*d*tan(c/2 + d*x/2) + 256000000*d) +
21656250*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)
/pi))*tan(c/2 + d*x/2)**5/(256000000*d*tan(c/2 + d*x/2)**6 + 921600000*d*t
an(c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)**4 + 2285568000*d*tan(c
/2 + d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2 + 921600000*d*tan(c/2 +
d*x/2) + 256000000*d) + 44034375*(atan(5*tan(c/2 + d*x/2)/4 + 3/4) + pi*fl
oor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(256000000*d*tan(c/2 + d
*x/2)**6 + 921600000*d*tan(c/2 + d*x/2)**5 + 1873920000*d*tan(c/2 + d*x/2)
**4 + 2285568000*d*tan(c/2 + d*x/2)**3 + 1873920000*d*tan(c/2 + d*x/2)**2
+ 921600000*d*tan(c/2 + d*x/2) + 256000000*d) + 53707500*(atan(5*tan(c/2 +
 d*x/2)/4 + 3/4) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**3/
(256000000*d*tan(c/2 + d*x/2)**6 + 921600000*d*tan(c/2 + d*x/2)**5 + 18739
20000*d*tan(c/2 + d*x/2)**4 + 2285568000*d*tan(c/2 + d*x/2)**3 + 187392000
0*d*tan(c/2 + d*x/2)**2 + 921600000*d*tan(c/2 + d*x/2) + 256000000*d) +...
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(96) = 192$.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.39

$$\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx$$

$$= \frac{\frac{36\left(\frac{403425\sin(dx+c)}{\cos(dx+c)+1}+\frac{672110\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{637794\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{373735\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{110925\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+142875\right)}{\frac{450\sin(dx+c)}{\cos(dx+c)+1}+\frac{915\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1116\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{915\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{450\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{125\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+125}+48125\arctan\left(\frac{5\sin(dx}{4(\cos(dx+}\right.}{2048000\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2048000*(36*(403425*sin(d*x + c)/(cos(d*x + c) + 1) + 672110*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 637794*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 37
3735*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 110925*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 142875)/(450*sin(d*x + c)/(cos(d*x + c) + 1) + 915*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 1116*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 91
5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 450*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 + 125*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 125) + 48125*arctan(5/4*
sin(d*x + c)/(cos(d*x + c) + 1) + 3/4))/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx$$

$$= \frac{48125\,dx + 48125\,c + \frac{72\left(110925\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+373735\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+637794\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+672110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+40342\right.}{\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)^3}}{4096000\,d}$$

input
```
integrate(1/(-5-3*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/4096000*(48125*d*x + 48125*c + 72*(110925*tan(1/2*d*x + 1/2*c)^5 + 37373
5*tan(1/2*d*x + 1/2*c)^4 + 637794*tan(1/2*d*x + 1/2*c)^3 + 672110*tan(1/2*
d*x + 1/2*c)^2 + 403425*tan(1/2*d*x + 1/2*c) + 142875)/(5*tan(1/2*d*x + 1/
2*c)^2 + 6*tan(1/2*d*x + 1/2*c) + 5)^3 + 96250*arctan(-(3*cos(d*x + c) + s
in(d*x + c) + 3)/(cos(d*x + c) - 3*sin(d*x + c) - 9)))/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.76

$$\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3}{4}\right)}{16384\,d} - \frac{385\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d}$$
$$+ \frac{\frac{39933\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2560000}+\frac{672723\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{12800000}+\frac{2870073\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000000}+\frac{604899\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6400000}+\frac{145233\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2560000}+\frac{10287}{512000}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{25}+\frac{1116\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{125}+\frac{183\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{25}+\frac{18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+1\right.}$$

input
```
int(1/(3*sin(c + d*x) + 5)^4,x)
```

output
```
(385*atan((5*tan(c/2 + (d*x)/2))/4 + 3/4))/(16384*d) - (385*(atan(tan(c/2
+ (d*x)/2)) - (d*x)/2))/(16384*d) + ((145233*tan(c/2 + (d*x)/2))/2560000 +
 (604899*tan(c/2 + (d*x)/2)^2)/6400000 + (2870073*tan(c/2 + (d*x)/2)^3)/32
000000 + (672723*tan(c/2 + (d*x)/2)^4)/12800000 + (39933*tan(c/2 + (d*x)/2
)^5)/2560000 + 10287/512000)/(d*((18*tan(c/2 + (d*x)/2))/5 + (183*tan(c/2
+ (d*x)/2)^2)/25 + (1116*tan(c/2 + (d*x)/2)^3)/125 + (183*tan(c/2 + (d*x)/
2)^4)/25 + (18*tan(c/2 + (d*x)/2)^5)/5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.33. $\int \frac{1}{(-5-3\sin(c+dx))^4}\,dx$

3.34 $\int \frac{1}{3+5\sin(c+dx)}\,dx$

3.34.1 Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{1}{3+5\sin(c+dx)}\,dx = -\frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

output
```
-1/4*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+1/4*ln(cos(1/2*d*x+1/2*
c)+3*sin(1/2*d*x+1/2*c))/d
```

3.34.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{3+5\sin(c+dx)}\,dx = -\frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

input
```
Integrate[(3 + 5*Sin[c + d*x])^(-1),x]
```

output
```
-1/4*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/d + Log[Cos[(c + d*x)/2] +
 3*Sin[(c + d*x)/2]]/(4*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\sin(c+dx)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{5\sin(c+dx)+3}dx$$

$$\Big\downarrow \text{3139}$$

$$\frac{2\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{1081}$$

$$\frac{6\int \left(\frac{1}{8\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{d}$$

input
```
Int[(3 + 5*Sin[c + d*x])^(-1),x]
```

output
```
(6*(-1/24*Log[3 + Tan[(c + d*x)/2]] + Log[1 + 3*Tan[(c + d*x)/2]]/24))/d
```

3.34.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.34.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{4}+\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}}{d}$	36
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{4}+\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}}{d}$	36
parallelrisch	$\frac{-\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9\right)+\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d}$	37
norman	$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{4d}+\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d}$	38
risch	$\frac{\ln\left(-\frac{4}{5}+\frac{3i}{5}+e^{i(dx+c)}\right)}{4d}-\frac{\ln\left(e^{i(dx+c)}+\frac{4}{5}+\frac{3i}{5}\right)}{4d}$	40

input
```
int(1/(3+5*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*ln(tan(1/2*d*x+1/2*c)+3)+1/4*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{3+5\sin(c+dx)}\,dx = -\frac{\log\left(4\cos\left(dx+c\right)+3\sin\left(dx+c\right)+5\right)-\log\left(-4\cos\left(dx+c\right)+3\sin\left(dx+c\right)+5\right)}{8\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(log(4*cos(d*x + c) + 3*sin(d*x + c) + 5) - log(-4*cos(d*x + c) + 3*s
in(d*x + c) + 5))/d
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \frac{1}{3+5\sin(c+dx)}\,dx = \begin{cases} -\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)}{4d} + \frac{\log\left(3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{5\sin(c)+3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3+5*sin(d*x+c)),x)
```

output
```
Piecewise((-log(tan(c/2 + d*x/2) + 3)/(4*d) + log(3*tan(c/2 + d*x/2) + 1)/
(4*d), Ne(d, 0)), (x/(5*sin(c) + 3), True))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{3+5\sin(c+dx)}\,dx = \frac{\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+1\right) - \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{4\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(log(3*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - log(sin(d*x + c)/(cos(d*
x + c) + 1) + 3))/d
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int \frac{1}{3+5\sin(c+dx)}\,dx = \frac{\log\left(\left|3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right|\right)}{4\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) - log(abs(tan(1/2*d*x + 1/2*c) +
 3)))/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.30

$$\int \frac{1}{3+5\sin(c+dx)}\,dx = -\frac{\operatorname{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{5}{4}\right)}{2\,d}$$

input
```
int(1/(5*sin(c + d*x) + 3),x)
```

output
```
-atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4)/(2*d)
```

3.35 $\int \frac{1}{(3+5\sin(c+dx))^2}\,dx$

3.35.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{(3+5\sin(c+dx))^2}\,dx = \frac{3\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{5\cos(c+dx)}{16d(3+5\sin(c+dx))}$$

output
```
3/64*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-3/64*ln(cos(1/2*d*x+1/2
*c)+3*sin(1/2*d*x+1/2*c))/d-5/16*cos(d*x+c)/d/(3+5*sin(d*x+c))
```

3.35.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.43

$$\int \frac{1}{(3+5\sin(c+dx))^2}\,dx = \frac{9\left(\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+20\sin\left(\frac{1}{2}(c+\right.}{192d}$$

input
```
Integrate[(3 + 5*Sin[c + d*x])^(-2),x]
```

output
```
(9*(Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 3*
Sin[(c + d*x)/2]]) + 20*Sin[(c + d*x)/2]*((3*Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^(-1) + 3/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])))/(192*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3143, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5\sin(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{16}\int -\frac{3}{5\sin(c+dx)+3}dx-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{3\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{1081}$$

$$-\frac{9\int \left(\frac{1}{8\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{8d}-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

input
```
Int[(3 + 5*Sin[c + d*x])^(-2),x]
```

output
```
(-9*(-1/24*Log[3 + Tan[(c + d*x)/2]] + Log[1 + 3*Tan[(c + d*x)/2]]/24))/(8
*d) - (5*Cos[c + d*x])/(16*d*(3 + 5*Sin[c + d*x]))
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.35.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}}{d}$
default	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}}{d}$
risch	$-\frac{3\,e^{i(dx+c)}+5i}{8d\left(5\,e^{2i(dx+c)}-5+6i e^{i(dx+c)}\right)}+\frac{3\ln\left(e^{i(dx+c)}+\frac{4}{5}+\frac{3i}{5}\right)}{64d}-\frac{3\ln\left(-\frac{4}{5}+\frac{3i}{5}+e^{i(dx+c)}\right)}{64d}$
norman	$\frac{-\frac{5}{8d}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}}{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{64d}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64d}$
parallelrisch	$\frac{45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)\sin(dx+c)-45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{3}\right)\sin(dx+c)-100\sin(dx+c)-60\cos(dx+c)+27\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{192d(3+5\sin(dx+c))}$

input
```
int(1/(3+5*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-5/16/(tan(1/2*d*x+1/2*c)+3)+3/64*ln(tan(1/2*d*x+1/2*c)+3)-5/48/(3*ta
n(1/2*d*x+1/2*c)+1)-3/64*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3+5\sin(c+dx))^2}\,dx = \frac{3\,(5\,\sin\left(dx+c\right)+3)\log\left(4\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)+5\right)-3\,(5\,\sin\left(dx+c\right)+3)\log\left(-4\,\cos\left(dx+c\right.\right.}{128\,(5\,d\sin\left(dx+c\right)+3\,d)}$$

input
```
integrate(1/(3+5*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/128*(3*(5*sin(d*x + c) + 3)*log(4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 3
*(5*sin(d*x + c) + 3)*log(-4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 40*cos(d
*x + c))/(5*d*sin(d*x + c) + 3*d)
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(78) = 156.

Time = 0.72 (sec) , antiderivative size = 466, normalized size of antiderivative = 5.30

$$\int \frac{1}{(3+5\sin(c+dx))^2}\,dx = \begin{cases} \frac{x}{\left(3-5\sin\left(2\operatorname{atan}\left(\frac{1}{3}\right)\right)\right)^2} \\ \frac{x}{\left(3-5\sin\left(2\operatorname{atan}\left(3\right)\right)\right)^2} \\ \frac{x}{\left(5\sin\left(c\right)+3\right)^2} \\ \frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d} + \frac{90\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d} + \frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right.} \end{cases}$$

input
```
integrate(1/(3+5*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(3 - 5*sin(2*atan(1/3)))**2, Eq(c, -d*x - 2*atan(1/3))), (x/(
3 - 5*sin(2*atan(3)))**2, Eq(c, -d*x - 2*atan(3))), (x/(5*sin(c) + 3)**2,
Eq(d, 0)), (27*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**2/(576*d*tan(c/
2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d) + 90*log(tan(c/2 + d*x/2)
 + 3)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2
) + 576*d) + 27*log(tan(c/2 + d*x/2) + 3)/(576*d*tan(c/2 + d*x/2)**2 + 192
0*d*tan(c/2 + d*x/2) + 576*d) - 27*log(3*tan(c/2 + d*x/2) + 1)*tan(c/2 + d
*x/2)**2/(576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d) - 9
0*log(3*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)**2
+ 1920*d*tan(c/2 + d*x/2) + 576*d) - 27*log(3*tan(c/2 + d*x/2) + 1)/(576*d
*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d) - 200*tan(c/2 + d*
x/2)/(576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d) - 120/(
576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d), True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \frac{1}{(3+5\sin(c+dx))^2}\,dx = -\frac{\frac{40\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{\frac{10\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+3} + 9\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+1\right) - 9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{192\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/192*(40*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 3)/(10*sin(d*x + c)/(cos(d
*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3) + 9*log(3*sin(d*
x + c)/(cos(d*x + c) + 1) + 1) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 3
))/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{(3+5\sin(c+dx))^2}\,dx = -\frac{\frac{40\,\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right)}{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}+9\,\log\left(\left|3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right|\right)}{192\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/192*(40*(5*tan(1/2*d*x + 1/2*c) + 3)/(3*tan(1/2*d*x + 1/2*c)^2 + 10*tan
(1/2*d*x + 1/2*c) + 3) + 9*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) - 9*log(ab
s(tan(1/2*d*x + 1/2*c) + 3)))/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \frac{1}{(3+5\sin(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{5}{4}\right)}{32\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{72}+\frac{5}{24}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1\right)}$$

input
```
int(1/(5*sin(c + d*x) + 3)^2,x)
```

output
```
(3*atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4))/(32*d) - ((25*tan(c/2 + (d*x)/2)
)/72 + 5/24)/(d*((10*tan(c/2 + (d*x)/2))/3 + tan(c/2 + (d*x)/2)^2 + 1))
```

3.36 $\int \frac{1}{(3+5\sin(c+dx))^3}\,dx$

3.36.1 Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{1}{(3+5\sin(c+dx))^3}\,dx = -\frac{43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5\cos(c+dx)}{32d(3+5\sin(c+dx))^2} + \frac{45\cos(c+dx)}{512d(3+5\sin(c+dx))}$$

output
```
-43/2048*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+43/2048*ln(cos(1/2*
d*x+1/2*c)+3*sin(1/2*d*x+1/2*c))/d-5/32*cos(d*x+c)/d/(3+5*sin(d*x+c))^2+45
/512*cos(d*x+c)/d/(3+5*sin(d*x+c))
```

3.36.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.59

$$\int \frac{1}{(3+5\sin(c+dx))^3}\,dx = \frac{-43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\quad}{(3\cos\left(\frac{1}{2}(c+d\right.}}{}$$

input
```
Integrate[(3 + 5*Sin[c + d*x])^(-3),x]
```

output
```
(-43*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 43*Log[Cos[(c + d*x)/2]
+ 3*Sin[(c + d*x)/2]] + 40/(3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 40/
(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])^2 + Sin[(c + d*x)/2]*(-60/(3*Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]) - 180/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/
2])))/(2048*d)
```

3.36.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5\sin(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{32}\int -\frac{6-5\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{32}\int \frac{6-5\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{32}\int \frac{6-5\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}-\frac{1}{16}\int -\frac{43}{5\sin(c+dx)+3}dx\right)-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{5\sin(c+dx)+3}dx+\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{5\sin(c+dx)+3}dx+\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \ 3139$$

$$\frac{1}{32}\left(\frac{43\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \ 1081$$

$$\frac{1}{32}\left(\frac{129\int \left(\frac{1}{8\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{32}\left(\frac{129\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{8d}+\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)-\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

input
```
Int[(3 + 5*Sin[c + d*x])^(-3),x]
```

output
```
(-5*Cos[c + d*x])/(32*d*(3 + 5*Sin[c + d*x])^2) + ((129*(-1/24*Log[3 + Tan
[(c + d*x)/2]] + Log[1 + 3*Tan[(c + d*x)/2]]/24))/(8*d) + (45*Cos[c + d*x]
)/(16*d*(3 + 5*Sin[c + d*x])))/32
```

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.36. $\int \frac{1}{(3+5\sin(c+dx))^3}\,dx$

3.36.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^2}-\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{2048}-\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{43}{}}{d}$
default	$\frac{\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^2}-\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{2048}-\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{43}{}}{d}$
risch	$\frac{387i\,e^{2i(dx+c)}+215\,e^{3i(dx+c)}-325\,e^{i(dx+c)}-225i}{256\left(5\,e^{2i(dx+c)}-5+6i\,e^{i(dx+c)}\right)^2 d}+\frac{43\ln\left(-\frac{4}{5}+\frac{3i}{5}+e^{i(dx+c)}\right)}{2048d}-\frac{43\ln\left(e^{i(dx+c)}+\frac{4}{5}+\frac{3i}{5}\right)}{2048d}$
norman	$\frac{\frac{55}{256d}+\frac{3245\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2304d}-\frac{125\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}+\frac{1225\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{768d}}{\left(3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^2}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{2048d}+\frac{43\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{2048d}$
parallelrisch	$\frac{(9675\cos(2dx+2c)-23220\sin(dx+c)-16641)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{3}\right)+(-9675\cos(2dx+2c)+23220\sin(dx+c)+16641)\ln}{18432d(-43+25\cos(2dx+2c)-6}$

input
```
int(1/(3+5*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(25/128/(tan(1/2*d*x+1/2*c)+3)^2-15/512/(tan(1/2*d*x+1/2*c)+3)-43/2048
*ln(tan(1/2*d*x+1/2*c)+3)-25/1152/(3*tan(1/2*d*x+1/2*c)+1)^2+155/4608/(3*t
an(1/2*d*x+1/2*c)+1)+43/2048*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{1}{(3+5\sin(c+dx))^3}\,dx = -\frac{43\left(25\cos\left(dx+c\right)^2-30\sin\left(dx+c\right)-34\right)\log\left(4\cos\left(dx+c\right)+3\sin\left(dx+c\right)+5\right)-43\left(25\cos\left(dx-\right.\right.}{4096\left(25\,d\cos\left(\right.\right.}$$

input
```
integrate(1/(3+5*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4096*(43*(25*cos(d*x + c)^2 - 30*sin(d*x + c) - 34)*log(4*cos(d*x + c)
+ 3*sin(d*x + c) + 5) - 43*(25*cos(d*x + c)^2 - 30*sin(d*x + c) - 34)*log(
-4*cos(d*x + c) + 3*sin(d*x + c) + 5) + 1800*cos(d*x + c)*sin(d*x + c) + 4
40*cos(d*x + c))/(25*d*cos(d*x + c)^2 - 30*d*sin(d*x + c) - 34*d)
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. $2(102) = 204$.

Time = 1.39 (sec) , antiderivative size = 1227, normalized size of antiderivative = 10.86

$$\int \frac{1}{(3+5\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3+5*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(3 - 5*sin(2*atan(1/3)))**3, Eq(c, -d*x - 2*atan(1/3))), (x/(
3 - 5*sin(2*atan(3)))**3, Eq(c, -d*x - 2*atan(3))), (x/(5*sin(c) + 3)**3,
Eq(d, 0)), (-3483*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**4/(165888*d*
tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 +
d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 23220*log(tan(c/2 + d
*x/2) + 3)*tan(c/2 + d*x/2)**3/(165888*d*tan(c/2 + d*x/2)**4 + 1105920*d*t
an(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 + 1105920*d*tan(c/2 + d
*x/2) + 165888*d) - 45666*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**2/(1
65888*d*tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*ta
n(c/2 + d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 23220*log(tan
(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)/(165888*d*tan(c/2 + d*x/2)**4 + 110592
0*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 + 1105920*d*tan(c/
2 + d*x/2) + 165888*d) - 3483*log(tan(c/2 + d*x/2) + 3)/(165888*d*tan(c/2
+ d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**
2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 3483*log(3*tan(c/2 + d*x/2) +
 1)*tan(c/2 + d*x/2)**4/(165888*d*tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/2
+ d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2) +
 165888*d) + 23220*log(3*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**3/(165888
*d*tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2
 + d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 45666*log(3*tan...
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.73

$$\int \frac{1}{(3+5\sin(c+dx))^3}\,dx = \frac{\frac{40\left(\frac{735\sin(dx+c)}{\cos(dx+c)+1}+\frac{649\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+99\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1}+\frac{118\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+9}+387\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+1\right)-387\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{18432\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/18432*(40*(735*sin(d*x + c)/(cos(d*x + c) + 1) + 649*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 99)/(60*sin(d*
x + c)/(cos(d*x + c) + 1) + 118*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + 9) + 387*log(3*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - 387*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 3))/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{1}{(3+5\sin(c+dx))^3}\,dx = -\frac{\frac{40\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-649\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-99\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^2}-387\log\left(\left|3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)+387\log\left(\left|\right.\right.}{18432\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/18432*(40*(75*tan(1/2*d*x + 1/2*c)^3 - 649*tan(1/2*d*x + 1/2*c)^2 - 735
*tan(1/2*d*x + 1/2*c) - 99)/(3*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1
/2*c) + 3)^2 - 387*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) + 387*log(abs(tan(
1/2*d*x + 1/2*c) + 3)))/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{1}{(3+5\sin(c+dx))^3}\,dx = \frac{-\frac{125\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6912}+\frac{3245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{20736}+\frac{1225\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6912}+\frac{55}{2304}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{118\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{9}+\frac{20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+1\right)} - \frac{43\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{5}{4}\right)}{1024\,d}$$

input
```
int(1/(5*sin(c + d*x) + 3)^3,x)
```

output
```
((1225*tan(c/2 + (d*x)/2))/6912 + (3245*tan(c/2 + (d*x)/2)^2)/20736 - (125
*tan(c/2 + (d*x)/2)^3)/6912 + 55/2304)/(d*((20*tan(c/2 + (d*x)/2))/3 + (11
8*tan(c/2 + (d*x)/2)^2)/9 + (20*tan(c/2 + (d*x)/2)^3)/3 + tan(c/2 + (d*x)/
2)^4 + 1)) - (43*atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4))/(1024*d)
```

3.36. $\int \frac{1}{(3+5\sin(c+dx))^3}\,dx$

3.37 $\int \frac{1}{(3+5\sin(c+dx))^4}\,dx$

3.37.1 Optimal result

Integrand size = 12, antiderivative size = 138

$$\int \frac{1}{(3+5\sin(c+dx))^4}\,dx = \frac{279\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{5\cos(c+dx)}{48d(3+5\sin(c+dx))^3} + \frac{25\cos(c+dx)}{512d(3+5\sin(c+dx))^2} - \frac{995\cos(c+dx)}{24576d(3+5\sin(c+dx))}$$

output
```
279/32768*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-279/32768*ln(cos(1
/2*d*x+1/2*c)+3*sin(1/2*d*x+1/2*c))/d-5/48*cos(d*x+c)/d/(3+5*sin(d*x+c))^3
+25/512*cos(d*x+c)/d/(3+5*sin(d*x+c))^2-995/24576*cos(d*x+c)/d/(3+5*sin(d*
x+c))
```

3.37.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.70

$$\int \frac{1}{(3+5\sin(c+dx))^4}\,dx$$
$$= \frac{2511\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2511\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{\cdots}{(3\cos(\frac{1}{2}(\cdots}}{\cdots}$$

input
```
Integrate[(3 + 5*Sin[c + d*x])^(-4),x]
```

output
```
(2511*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2511*Log[Cos[(c + d*x)/
2] + 3*Sin[(c + d*x)/2]] - 2320/(3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
+ 720/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])^2 + 20*Sin[(c + d*x)/2]*(80/
(3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 199/(3*Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]) + 240/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])^3 + 597/(Cos[(
c + d*x)/2] + 3*Sin[(c + d*x)/2])))/(294912*d)
```

3.37.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(c+dx)+3)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(5\sin(c+dx)+3)^4}dx$$
$$\Big\downarrow \text{3143}$$
$$\frac{1}{48}\int -\frac{9-10\sin(c+dx)}{(5\sin(c+dx)+3)^3}dx-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{48}\int \frac{9-10\sin(c+dx)}{(5\sin(c+dx)+3)^3}dx-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{48}\int \frac{9-10\sin(c+dx)}{(5\sin(c+dx)+3)^3}dx-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 3233$$

$$\frac{1}{48}\left(\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}-\frac{1}{32}\int -\frac{154-75\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 3233$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{1}{16}\int -\frac{837}{5\sin(c+dx)+3}dx-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{837}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{837}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 3139$$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{837\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+d}{32d(5\sin(c+dx}\right.$$
$$\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \; 1081$$

3.37. $\int \frac{1}{(3+5\sin(c+dx))^4}\,dx$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{2511\int\left(\frac{1}{8(3\tan(\frac{1}{2}(c+dx))+1)}-\frac{1}{24(\tan(\frac{1}{2}(c+dx))+3)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{}{32d(}\right.$$

$$\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$\downarrow$ 2009

$$\frac{1}{48}\left(\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}+\frac{1}{32}\left(-\frac{2511\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{8d}-\frac{9}{16d(}\right.\right.$$

$$\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

input
```
Int[(3 + 5*Sin[c + d*x])^(-4),x]
```

output
```
(-5*Cos[c + d*x])/(48*d*(3 + 5*Sin[c + d*x])^3) + ((75*Cos[c + d*x])/(32*d
*(3 + 5*Sin[c + d*x])^2) + ((-2511*(-1/24*Log[3 + Tan[(c + d*x)/2]] + Log[
1 + 3*Tan[(c + d*x)/2]]/24))/(8*d) - (995*Cos[c + d*x])/(16*d*(3 + 5*Sin[c
 + d*x])))/32)/48
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.37.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}+\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{d}$
default	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}+\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{d}$
risch	$-\frac{-111042\,\mathrm{e}^{3i(dx+c)}+62775i\mathrm{e}^{4i(dx+c)}-119310i\mathrm{e}^{2i(dx+c)}+20925\,\mathrm{e}^{5i(dx+c)}+68625\,\mathrm{e}^{i(dx+c)}+24875i}{12288\left(5\,\mathrm{e}^{2i(dx+c)}-5+6i\mathrm{e}^{i(dx+c)}\right)^{3}d}+\frac{279\ln\left(\mathrm{e}^{i(dx+c}\right.}{32768}$
norman	$\frac{-\frac{7915}{12288d}-\frac{63425\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12288d}-\frac{3047275\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{165888d}-\frac{15725\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12288d}-\frac{296245\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{18432d}-\frac{270245\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{36864d}}{\left(3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}$
parallelrisch	$\frac{(-10169550\cos(2dx+2c)+20678085\sin(dx+c)-2824875\sin(3dx+3c)+12610242)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{3}\right)+(10169550\cos}{}$

input
```
int(1/(3+5*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-125/768/(tan(1/2*d*x+1/2*c)+3)^3+75/1024/(tan(1/2*d*x+1/2*c)+3)^2-34
5/8192/(tan(1/2*d*x+1/2*c)+3)+279/32768*ln(tan(1/2*d*x+1/2*c)+3)-125/20736
/(3*tan(1/2*d*x+1/2*c)+1)^3+275/27648/(3*tan(1/2*d*x+1/2*c)+1)^2-3505/2211
84/(3*tan(1/2*d*x+1/2*c)+1)-279/32768*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\int \frac{1}{(3+5\sin(c+dx))^4}\,dx =$$
$$-\frac{199000\cos(dx+c)^3 - 837\left(225\cos(dx+c)^2 + 5\left(25\cos(dx+c)^2 - 52\right)\sin(dx+c) - 252\right)\log(4\,\mathrm{co}}{}$$

input
```
integrate(1/(3+5*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/196608*(199000*cos(d*x + c)^3 - 837*(225*cos(d*x + c)^2 + 5*(25*cos(d*x
 + c)^2 - 52)*sin(d*x + c) - 252)*log(4*cos(d*x + c) + 3*sin(d*x + c) + 5)
 + 837*(225*cos(d*x + c)^2 + 5*(25*cos(d*x + c)^2 - 52)*sin(d*x + c) - 252
)*log(-4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 190800*cos(d*x + c)*sin(d*x
+ c) - 262320*cos(d*x + c))/(225*d*cos(d*x + c)^2 + 5*(25*d*cos(d*x + c)^2
 - 52*d)*sin(d*x + c) - 252*d)
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2356 vs. $2(126) = 252$.

Time = 3.13 (sec) , antiderivative size = 2356, normalized size of antiderivative = 17.07

$$\int \frac{1}{(3+5\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3+5*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(3 - 5*sin(2*atan(1/3)))**4, Eq(c, -d*x - 2*atan(1/3))), (x/(
3 - 5*sin(2*atan(3)))**4, Eq(c, -d*x - 2*atan(3))), (x/(5*sin(c) + 3)**4,
Eq(d, 0)), (610173*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**6/(71663616
*d*tan(c/2 + d*x/2)**6 + 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*ta
n(c/2 + d*x/2)**4 + 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/
2 + d*x/2)**2 + 716636160*d*tan(c/2 + d*x/2) + 71663616*d) + 6101730*log(t
an(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**5/(71663616*d*tan(c/2 + d*x/2)**6 +
 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 + 4087
480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 + 71663616
0*d*tan(c/2 + d*x/2) + 71663616*d) + 22169619*log(tan(c/2 + d*x/2) + 3)*ta
n(c/2 + d*x/2)**4/(71663616*d*tan(c/2 + d*x/2)**6 + 716636160*d*tan(c/2 +
d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 + 4087480320*d*tan(c/2 + d*x/
2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 + 716636160*d*tan(c/2 + d*x/2) +
71663616*d) + 34802460*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**3/(7166
3616*d*tan(c/2 + d*x/2)**6 + 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*
d*tan(c/2 + d*x/2)**4 + 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*ta
n(c/2 + d*x/2)**2 + 716636160*d*tan(c/2 + d*x/2) + 71663616*d) + 22169619*
log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**2/(71663616*d*tan(c/2 + d*x/2)
**6 + 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 +
 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 + ...
```

3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(124) = 248$.

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.99

$$\int \frac{1}{(3+5\sin(c+dx))^4}\,dx =$$

$$-\frac{\frac{40\left(\frac{342495\sin(dx+c)}{\cos(dx+c)+1}+\frac{1066482\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1218910\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{486441\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84915\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+42741\right)}{\frac{270\sin(dx+c)}{\cos(dx+c)+1}+\frac{981\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1540\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{981\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{270\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{27\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+27}+22599\log\left(\frac{3\sin(dx+c}{\cos(dx+c)+}\right.}{2654208\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/2654208*(40*(342495*sin(d*x + c)/(cos(d*x + c) + 1) + 1066482*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 1218910*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 486441*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 84915*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5 + 42741)/(270*sin(d*x + c)/(cos(d*x + c) + 1) + 981*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9
81*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 270*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 27*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 27) + 22599*log(3*sin(d*
x + c)/(cos(d*x + c) + 1) + 1) - 22599*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 3))/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{1}{(3+5\sin(c+dx))^4}\,dx =$$

$$-\frac{\frac{40\left(84915\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+486441\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1218910\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1066482\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+342495\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+42741\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^3}}{2654208\,d}$$

input
```
integrate(1/(3+5*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/2654208*(40*(84915*tan(1/2*d*x + 1/2*c)^5 + 486441*tan(1/2*d*x + 1/2*c)
^4 + 1218910*tan(1/2*d*x + 1/2*c)^3 + 1066482*tan(1/2*d*x + 1/2*c)^2 + 342
495*tan(1/2*d*x + 1/2*c) + 42741)/(3*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d
*x + 1/2*c) + 3)^3 + 22599*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) - 22599*lo
g(abs(tan(1/2*d*x + 1/2*c) + 3)))/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.22

$$\int \frac{1}{(3+5\sin(c+dx))^4}\,dx = \frac{279\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{5}{4}\right)}{16384\,d}$$

$$-\frac{\frac{15725\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{331776}+\frac{270245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{995328}+\frac{3047275\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4478976}+\frac{296245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{497664}+\frac{63425\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{331776}+\frac{79}{331}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{1540\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{27}+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+10\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(1/(5*sin(c + d*x) + 3)^4,x)
```

output
```
(279*atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4))/(16384*d) - ((63425*tan(c/2 +
(d*x)/2))/331776 + (296245*tan(c/2 + (d*x)/2)^2)/497664 + (3047275*tan(c/2
 + (d*x)/2)^3)/4478976 + (270245*tan(c/2 + (d*x)/2)^4)/995328 + (15725*tan
(c/2 + (d*x)/2)^5)/331776 + 7915/331776)/(d*(10*tan(c/2 + (d*x)/2) + (109*
tan(c/2 + (d*x)/2)^2)/3 + (1540*tan(c/2 + (d*x)/2)^3)/27 + (109*tan(c/2 +
(d*x)/2)^4)/3 + 10*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.37. $\int \frac{1}{(3+5\sin(c+dx))^4}\,dx$

3.38 $\int \frac{1}{3-5\sin(c+dx)}\,dx$

3.38.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{1}{3-5\sin(c+dx)}\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

output
```
-1/4*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d+1/4*ln(3*cos(1/2*d*x+1/
2*c)-sin(1/2*d*x+1/2*c))/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{3-5\sin(c+dx)}\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

input
```
Integrate[(3 - 5*Sin[c + d*x])^(-1),x]
```

output
```
-1/4*Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]]/d + Log[3*Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]]/(4*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3-5\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{3-5\sin(c+dx)}dx$$

$$\Big\downarrow \; 3139$$

$$\frac{2\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \; 1081$$

$$\frac{6\int \left(\frac{1}{8\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{24\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{6\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

input
```
Int[(3 - 5*Sin[c + d*x])^(-1),x]
```

output
```
(6*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24))/d
```

3.38.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{4}-\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}}{d}$	36
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{4}-\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}}{d}$	36
parallelrisch	$\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-9\right)-\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$	37
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{4d}-\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$	38
risch	$-\frac{\ln\left(-\frac{4}{5}-\frac{3i}{5}+e^{i(dx+c)}\right)}{4d}+\frac{\ln\left(e^{i(dx+c)}+\frac{4}{5}-\frac{3i}{5}\right)}{4d}$	40

input
```
int(1/(3-5*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*ln(tan(1/2*d*x+1/2*c)-3)-1/4*ln(3*tan(1/2*d*x+1/2*c)-1))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{1}{3-5\sin(c+dx)}\,dx = \frac{\log\left(4\,\cos\left(dx+c\right)-3\,\sin\left(dx+c\right)+5\right)-\log\left(-4\,\cos\left(dx+c\right)-3\,\sin\left(dx+c\right)+5\right)}{8\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(log(4*cos(d*x + c) - 3*sin(d*x + c) + 5) - log(-4*cos(d*x + c) - 3*si
n(d*x + c) + 5))/d
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{1}{3-5\sin(c+dx)}\,dx = \begin{cases} \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)}{4d} - \frac{\log\left(3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{4d} & \text{for } d\neq 0 \\ \frac{x}{3-5\sin(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3-5*sin(d*x+c)),x)
```

output
```
Piecewise(((log(tan(c/2 + d*x/2) - 3)/(4*d) - log(3*tan(c/2 + d*x/2) - 1)/(
4*d), Ne(d, 0)), (x/(3 - 5*sin(c)), True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{1}{3-5\sin(c+dx)}\,dx = -\frac{\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-1\right)-\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{4\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(log(3*sin(d*x + c)/(cos(d*x + c) + 1) - 1) - log(sin(d*x + c)/(cos(d
*x + c) + 1) - 3))/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{1}{3-5\sin(c+dx)}\,dx = -\frac{\log\left(\left|3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\right|\right)}{4\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) - log(abs(tan(1/2*d*x + 1/2*c)
- 3)))/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.29

$$\int \frac{1}{3-5\sin(c+dx)}\,dx = -\frac{\operatorname{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{5}{4}\right)}{2\,d}$$

input
```
int(-1/(5*sin(c + d*x) - 3),x)
```

output
```
-atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4)/(2*d)
```

3.39 $\int \frac{1}{(3-5\sin(c+dx))^2}\,dx$

3.39.1 Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(3-5\sin(c+dx))^2}\,dx = \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}$$

output
```
3/64*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d-3/64*ln(3*cos(1/2*d*x+1
/2*c)-sin(1/2*d*x+1/2*c))/d+5/16*cos(d*x+c)/d/(3-5*sin(d*x+c))
```

3.39.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

$$\int \frac{1}{(3-5\sin(c+dx))^2}\,dx = \frac{9\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+20\left(\frac{\ldots}{\cos\left(\frac{1}{2}(c+dx)\right)\ldots}\right.}{192d}$$

input
```
Integrate[(3 - 5*Sin[c + d*x])^(-2),x]
```

output
```
(9*(Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]] - Log[3*Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]]) + 20*(3/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]) + (3*Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^(-1))*Sin[(c + d*x)/2])/(192*d)
```

3.39.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3143, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-5\sin(c+dx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(3-5\sin(c+dx))^2}dx$$

$$\Big\downarrow \ 3143$$

$$\frac{1}{16}\int -\frac{3}{3-5\sin(c+dx)}dx+\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}$$

$$\Big\downarrow \ 27$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{3}{16}\int \frac{1}{3-5\sin(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{3}{16}\int \frac{1}{3-5\sin(c+dx)}dx$$

$$\Big\downarrow \ 3139$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{3\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}$$

$$\Big\downarrow \ 1081$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{9\int \left(\frac{1}{8\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{24\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}$$

$$\Big\downarrow \ 2009$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{9\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}$$

input
```
Int[(3 - 5*Sin[c + d*x])^(-2),x]
```

output
```
(-9*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24))/(8
*d) + (5*Cos[c + d*x])/(16*d*(3 - 5*Sin[c + d*x]))
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.39.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
default	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
risch	$\frac{3\,e^{i(dx+c)}-5i}{8d\left(5\,e^{2i(dx+c)}-5-6ie^{i(dx+c)}\right)}+\frac{3\ln\left(-\frac{4}{5}-\frac{3i}{5}+e^{i(dx+c)}\right)}{64d}-\frac{3\ln\left(e^{i(dx+c)}+\frac{4}{5}-\frac{3i}{5}\right)}{64d}$
norman	$\frac{\frac{5}{8d}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}}{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{64d}+\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64d}$
parallelrisch	$\frac{-45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)\sin(dx+c)+45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3}\right)\sin(dx+c)+27\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)-27\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{192d(-3+5\sin(dx+c))}$

input

```
int(1/(3-5*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-5/16/(tan(1/2*d*x+1/2*c)-3)-3/64*ln(tan(1/2*d*x+1/2*c)-3)-5/48/(3*ta
n(1/2*d*x+1/2*c)-1)+3/64*ln(3*tan(1/2*d*x+1/2*c)-1))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{1}{(3-5\sin(c+dx))^2}\,dx =$$
$$-\frac{3\,(5\,\sin{(dx+c)}-3)\log{(4\,\cos{(dx+c)}-3\,\sin{(dx+c)}+5)}-3\,(5\,\sin{(dx+c)}-3)\log{(-4\,\cos{(dx-}}{128\,(5\,d\sin{(dx+c)}-3\,d)}$$

input

```
integrate(1/(3-5*sin(d*x+c))^2,x, algorithm="fricas")
```

output

```
-1/128*(3*(5*sin(d*x + c) - 3)*log(4*cos(d*x + c) - 3*sin(d*x + c) + 5) -
3*(5*sin(d*x + c) - 3)*log(-4*cos(d*x + c) - 3*sin(d*x + c) + 5) + 40*cos(
d*x + c))/(5*d*sin(d*x + c) - 3*d)
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(78) = 156$.

Time = 0.75 (sec) , antiderivative size = 462, normalized size of antiderivative = 5.13

$$\int \frac{1}{(3-5\sin(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{x}{\left(3-5\sin\left(2\operatorname{atan}\left(\frac{1}{3}\right)\right)\right)^2} \\ \frac{x}{\left(3-5\sin\left(2\operatorname{atan}\left(3\right)\right)\right)^2} \\ \frac{x}{\left(3-5\sin\left(c\right)\right)^2} \\ -\frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d}+\frac{90\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d}-\frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1920d\tan\left(\frac{c}{2}+ \right.} \end{cases}$$

input
```
integrate(1/(3-5*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(3 - 5*sin(2*atan(1/3)))**2, Eq(c, -d*x + 2*atan(1/3))), (x/(
3 - 5*sin(2*atan(3)))**2, Eq(c, -d*x + 2*atan(3))), (x/(3 - 5*sin(c))**2,
Eq(d, 0)), (-27*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**2/(576*d*tan(c
/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d) + 90*log(tan(c/2 + d*x/2
) - 3)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/
2) + 576*d) - 27*log(tan(c/2 + d*x/2) - 3)/(576*d*tan(c/2 + d*x/2)**2 - 19
20*d*tan(c/2 + d*x/2) + 576*d) + 27*log(3*tan(c/2 + d*x/2) - 1)*tan(c/2 +
d*x/2)**2/(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d) -
90*log(3*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)**2
 - 1920*d*tan(c/2 + d*x/2) + 576*d) + 27*log(3*tan(c/2 + d*x/2) - 1)/(576*
d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d) - 200*tan(c/2 + d
*x/2)/(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d) + 120/
(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d), True))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{1}{(3-5\sin(c+dx))^2}\,dx = \frac{\frac{40\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{\frac{10\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-3}+9\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-1\right)-9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{192\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/192*(40*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 3)/(10*sin(d*x + c)/(cos(d*
x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3) + 9*log(3*sin(d*x
 + c)/(cos(d*x + c) + 1) - 1) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 3)
)/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3-5\sin(c+dx))^2}\,dx = -\frac{\frac{40\,(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3)}{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3} - 9\,\log\left(\left|3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + 9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\right|\right)}{192\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/192*(40*(5*tan(1/2*d*x + 1/2*c) - 3)/(3*tan(1/2*d*x + 1/2*c)^2 - 10*tan
(1/2*d*x + 1/2*c) + 3) - 9*log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) + 9*log(ab
s(tan(1/2*d*x + 1/2*c) - 3)))/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int \frac{1}{(3-5\sin(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{5}{4}\right)}{32\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{72}-\frac{5}{24}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1\right)}$$

input
```
int(1/(5*sin(c + d*x) - 3)^2,x)
```

output
```
(3*atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4))/(32*d) - ((25*tan(c/2 + (d*x)/2)
)/72 - 5/24)/(d*(tan(c/2 + (d*x)/2)^2 - (10*tan(c/2 + (d*x)/2))/3 + 1))
```

3.40 $\int \frac{1}{(3-5\sin(c+dx))^3}\,dx$

3.40.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx = -\frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2} - \frac{45\cos(c+dx)}{512d(3-5\sin(c+dx))}$$

output
```
-43/2048*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d+43/2048*ln(3*cos(1/
2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d+5/32*cos(d*x+c)/d/(3-5*sin(d*x+c))^2-45
/512*cos(d*x+c)/d/(3-5*sin(d*x+c))
```

3.40.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.60

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx = \frac{-43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)+43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\cdots}{(\cos(\frac{1}{2}(c+dx\cdots}}{\cdots}$$

input
```
Integrate[(3 - 5*Sin[c + d*x])^(-3),x]
```

output
```
(-43*Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]] + 43*Log[3*Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 40/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2])^2 + (-1
80/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]) - 60/(3*Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]))*Sin[(c + d*x)/2] - 40/(-3*Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2)/(2048*d)
```

3.40.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3-5\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{32}\int -\frac{5\sin(c+dx)+6}{(3-5\sin(c+dx))^2}dx+\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}-\frac{1}{32}\int \frac{5\sin(c+dx)+6}{(3-5\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}-\frac{1}{32}\int \frac{5\sin(c+dx)+6}{(3-5\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{43}{3-5\sin(c+dx)}dx-\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)+\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{3-5\sin(c+dx)}dx-\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)+\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{3-5\sin(c+dx)}dx-\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)+\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{32}\left(\frac{43\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)+\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{32}\left(\frac{129\int \left(\frac{1}{8\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{24\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)+\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}+\frac{1}{32}\left(\frac{129\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}-\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)$$

input
```
Int[(3 - 5*Sin[c + d*x])^(-3),x]
```

output
```
((129*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24))/
(8*d) - (45*Cos[c + d*x])/(16*d*(3 - 5*Sin[c + d*x])))/32 + (5*Cos[c + d*x
])/(32*d*(3 - 5*Sin[c + d*x])^2)
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.40. $\int \frac{1}{(3-5\sin(c+dx))^3}\,dx$

3.40.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}-\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{2048}+\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-}{d}$
default	$\frac{-\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}-\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{2048}+\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-}{d}$
risch	$-\frac{-387i\,e^{2i(dx+c)}+215\,e^{3i(dx+c)}-325\,e^{i(dx+c)}+225i}{256\left(5\,e^{2i(dx+c)}-5-6i\,e^{i(dx+c)}\right)^{2}d}+\frac{43\ln\left(e^{i(dx+c)}+\frac{4}{5}-\frac{3i}{5}\right)}{2048d}-\frac{43\ln\left(-\frac{4}{5}-\frac{3i}{5}+e^{i(dx+c)}\right)}{2048d}$
norman	$\frac{-\frac{55}{256d}-\frac{3245\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2304d}+\frac{1225\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{768d}-\frac{125\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}}{\left(3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{2048d}-\frac{43\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2048d}$
parallelrisch	$\frac{(-9675\cos(2dx+2c)-23220\sin(dx+c)+16641)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3}\right)+(9675\cos(2dx+2c)+23220\sin(dx+c)-16641)\ln}{18432d(-43+25\cos(2dx+2c)+6}$

input
```
int(1/(3-5*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-25/128/(tan(1/2*d*x+1/2*c)-3)^2-15/512/(tan(1/2*d*x+1/2*c)-3)+43/204
8*ln(tan(1/2*d*x+1/2*c)-3)+25/1152/(3*tan(1/2*d*x+1/2*c)-1)^2+155/4608/(3*
tan(1/2*d*x+1/2*c)-1)-43/2048*ln(3*tan(1/2*d*x+1/2*c)-1))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx$$
$$=\frac{43\left(25\cos\left(dx+c\right)^{2}+30\sin\left(dx+c\right)-34\right)\log\left(4\cos\left(dx+c\right)-3\sin\left(dx+c\right)+5\right)-43\left(25\cos\left(dx+c\right.\right.}{4096\left(25\,d\cos\left(dx\right.\right.}$$

input
```
integrate(1/(3-5*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4096*(43*(25*cos(d*x + c)^2 + 30*sin(d*x + c) - 34)*log(4*cos(d*x + c) -
 3*sin(d*x + c) + 5) - 43*(25*cos(d*x + c)^2 + 30*sin(d*x + c) - 34)*log(-
4*cos(d*x + c) - 3*sin(d*x + c) + 5) - 1800*cos(d*x + c)*sin(d*x + c) + 44
0*cos(d*x + c))/(25*d*cos(d*x + c)^2 + 30*d*sin(d*x + c) - 34*d)
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(102) = 204$.

Time = 1.43 (sec) , antiderivative size = 1224, normalized size of antiderivative = 10.64

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3-5*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(3 - 5*sin(2*atan(1/3)))**3, Eq(c, -d*x + 2*atan(1/3))), (x/(
3 - 5*sin(2*atan(3)))**3, Eq(c, -d*x + 2*atan(3))), (x/(3 - 5*sin(c))**3,
Eq(d, 0)), (3483*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**4/(165888*d*t
an(c/2 + d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d
*x/2)**2 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 23220*log(tan(c/2 + d*
x/2) - 3)*tan(c/2 + d*x/2)**3/(165888*d*tan(c/2 + d*x/2)**4 - 1105920*d*ta
n(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 - 1105920*d*tan(c/2 + d*
x/2) + 165888*d) + 45666*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**2/(16
5888*d*tan(c/2 + d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan
(c/2 + d*x/2)**2 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 23220*log(tan(
c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)/(165888*d*tan(c/2 + d*x/2)**4 - 1105920
*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 - 1105920*d*tan(c/2
 + d*x/2) + 165888*d) + 3483*log(tan(c/2 + d*x/2) - 3)/(165888*d*tan(c/2 +
 d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2
 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 3483*log(3*tan(c/2 + d*x/2) -
1)*tan(c/2 + d*x/2)**4/(165888*d*tan(c/2 + d*x/2)**4 - 1105920*d*tan(c/2 +
 d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 - 1105920*d*tan(c/2 + d*x/2) +
165888*d) + 23220*log(3*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**3/(165888*
d*tan(c/2 + d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2
+ d*x/2)**2 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 45666*log(3*tan(...
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.70

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{40\left(\frac{735\sin(dx+c)}{\cos(dx+c)+1}-\frac{649\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-99\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1}-\frac{118\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-9}+387\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-1\right)-387\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{18432\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/18432*(40*(735*sin(d*x + c)/(cos(d*x + c) + 1) - 649*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 99)/(60*sin(d
*x + c)/(cos(d*x + c) + 1) - 118*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 9) + 387*log(3*sin(d*x + c)/(cos(d*x + c) + 1) - 1) - 387*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 3))/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx = -\frac{\frac{40\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+649\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+99\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^2}+387\log\left(\left|3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-387\log\left(\left|\right.\right.}{18432\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/18432*(40*(75*tan(1/2*d*x + 1/2*c)^3 + 649*tan(1/2*d*x + 1/2*c)^2 - 735
*tan(1/2*d*x + 1/2*c) + 99)/(3*tan(1/2*d*x + 1/2*c)^2 - 10*tan(1/2*d*x + 1
/2*c) + 3)^2 + 387*log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) - 387*log(abs(tan(
1/2*d*x + 1/2*c) - 3)))/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3-5\sin(c+dx))^3}\,dx = -\frac{43\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{5}{4}\right)}{1024\,d} - \frac{\frac{125\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6912}+\frac{3245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{20736}-\frac{1225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6912}+\frac{55}{2304}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{118\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{9}-\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1\right)}$$

input
```
int(-1/(5*sin(c + d*x) - 3)^3,x)
```

output
```
- (43*atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4))/(1024*d) - ((3245*tan(c/2 + (
d*x)/2)^2)/20736 - (1225*tan(c/2 + (d*x)/2))/6912 + (125*tan(c/2 + (d*x)/2
)^3)/6912 + 55/2304)/(d*((118*tan(c/2 + (d*x)/2)^2)/9 - (20*tan(c/2 + (d*x
)/2))/3 - (20*tan(c/2 + (d*x)/2)^3)/3 + tan(c/2 + (d*x)/2)^4 + 1))
```

3.40. $\int \frac{1}{(3-5\sin(c+dx))^3}\,dx$

3.41 $\int \frac{1}{(3-5\sin(c+dx))^4}\,dx$

3.41.1 Optimal result

Integrand size = 12, antiderivative size = 140

$$\int \frac{1}{(3-5\sin(c+dx))^4}\,dx = \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3} - \frac{25\cos(c+dx)}{512d(3-5\sin(c+dx))^2} + \frac{995\cos(c+dx)}{24576d(3-5\sin(c+dx))}$$

output
```
279/32768*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d-279/32768*ln(3*cos
(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d+5/48*cos(d*x+c)/d/(3-5*sin(d*x+c))^3
-25/512*cos(d*x+c)/d/(3-5*sin(d*x+c))^2+995/24576*cos(d*x+c)/d/(3-5*sin(d*
x+c))
```

3.41.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\int \frac{1}{(3-5\sin(c+dx))^4}\,dx$$
$$= \frac{2511\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)-2511\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{\quad}{(\cos\left(\frac{1}{2}(c-\right.}}{}$$

input
```
Integrate[(3 - 5*Sin[c + d*x])^(-4),x]
```

output
```
(2511*Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]] - 2511*Log[3*Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - 720/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2])^2 +
 20*(240/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2])^3 + 597/(Cos[(c + d*x)/2]
 - 3*Sin[(c + d*x)/2]) + 80/(3*Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + 19
9/(3*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))*Sin[(c + d*x)/2] + 2320/(-3*Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(294912*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-5\sin(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(3-5\sin(c+dx))^4}dx$$
$$\Big\downarrow \text{3143}$$
$$\frac{1}{48}\int -\frac{10\sin(c+dx)+9}{(3-5\sin(c+dx))^3}dx+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$
$$\Big\downarrow \text{25}$$
$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}-\frac{1}{48}\int \frac{10\sin(c+dx)+9}{(3-5\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}-\frac{1}{48}\int\frac{10\sin(c+dx)+9}{(3-5\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(-\frac{1}{32}\int-\frac{75\sin(c+dx)+154}{(3-5\sin(c+dx))^2}dx-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{48}\left(\frac{1}{32}\int\frac{75\sin(c+dx)+154}{(3-5\sin(c+dx))^2}dx-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{1}{32}\int\frac{75\sin(c+dx)+154}{(3-5\sin(c+dx))^2}dx-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{1}{16}\int-\frac{837}{3-5\sin(c+dx)}dx+\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{837}{16}\int\frac{1}{3-5\sin(c+dx)}dx\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{837}{16}\int\frac{1}{3-5\sin(c+dx)}dx\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{837\int\frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx)}\right.$$
$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{1081}$$

3.41. $\int\frac{1}{(3-5\sin(c+dx))^4}\,dx$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{2511\int\left(\frac{1}{8\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{24\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}\right)-\frac{75}{32d(3}\right.$$

$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}+$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{2511\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}\right)-\frac{7}{32d(3}\right.$$

input
```
Int[(3 - 5*Sin[c + d*x])^(-4),x]
```

output
```
(((-2511*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24
))/(8*d) + (995*Cos[c + d*x])/(16*d*(3 - 5*Sin[c + d*x])))/32 - (75*Cos[c
+ d*x])/(32*d*(3 - 5*Sin[c + d*x])^2))/48 + (5*Cos[c + d*x])/(48*d*(3 - 5*
Sin[c + d*x])^3)
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.41.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{3}}-\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}} \cdots}{d}$
default	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{3}}-\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}} \cdots}{d}$
risch	$\frac{-111042\,e^{3i(dx+c)}-62775i e^{4i(dx+c)}+119310i e^{2i(dx+c)}+20925\,e^{5i(dx+c)}+68625\,e^{i(dx+c)}-24875i}{12288\left(5\,e^{2i(dx+c)}-5-6i e^{i(dx+c)}\right)^{3}d}+\frac{279\ln\left(-\frac{4}{5}-\frac{3i}{5}+\cdots\right.}{32768d}$
norman	$\frac{\frac{7915}{12288d}-\frac{15725\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12288d}-\frac{3047275\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{165888d}-\frac{63425\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12288d}+\frac{296245\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{18432d}+\frac{270245\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{36864d} \cdots}{\left(3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}$
parallelrisch	$\frac{\left(10169550\cos(2dx+2c)+20678085\sin(dx+c)-2824875\sin(3dx+3c)-12610242\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3}\right)+\left(-10169550\cos \cdots\right.}{\cdots}$

input
```
int(1/(3-5*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-125/768/(tan(1/2*d*x+1/2*c)-3)^3-75/1024/(tan(1/2*d*x+1/2*c)-3)^2-34
5/8192/(tan(1/2*d*x+1/2*c)-3)-279/32768*ln(tan(1/2*d*x+1/2*c)-3)-125/20736
/(3*tan(1/2*d*x+1/2*c)-1)^3-275/27648/(3*tan(1/2*d*x+1/2*c)-1)^2-3505/2211
84/(3*tan(1/2*d*x+1/2*c)-1)+279/32768*ln(3*tan(1/2*d*x+1/2*c)-1))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int \frac{1}{(3-5\sin(c+dx))^4}\,dx$$
$$= \frac{199000\,\cos\left(dx+c\right)^3-837\left(225\,\cos\left(dx+c\right)^2-5\left(25\,\cos\left(dx+c\right)^2-52\right)\sin\left(dx+c\right)-252\right)\log\left(4\,\cos\left(\right.\right.}{}$$

input
```
integrate(1/(3-5*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/196608*(199000*cos(d*x + c)^3 - 837*(225*cos(d*x + c)^2 - 5*(25*cos(d*x
+ c)^2 - 52)*sin(d*x + c) - 252)*log(4*cos(d*x + c) - 3*sin(d*x + c) + 5)
+ 837*(225*cos(d*x + c)^2 - 5*(25*cos(d*x + c)^2 - 52)*sin(d*x + c) - 252)
*log(-4*cos(d*x + c) - 3*sin(d*x + c) + 5) + 190800*cos(d*x + c)*sin(d*x +
 c) - 262320*cos(d*x + c))/(225*d*cos(d*x + c)^2 - 5*(25*d*cos(d*x + c)^2
- 52*d)*sin(d*x + c) - 252*d)
```

3.41.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2353 vs. $2(126) = 252$.

Time = 3.15 (sec) , antiderivative size = 2353, normalized size of antiderivative = 16.81

$$\int \frac{1}{(3-5\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3-5*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(3 - 5*sin(2*atan(1/3)))**4, Eq(c, -d*x + 2*atan(1/3))), (x/(
3 - 5*sin(2*atan(3)))**4, Eq(c, -d*x + 2*atan(3))), (x/(3 - 5*sin(c))**4,
Eq(d, 0)), (-610173*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**6/(7166361
6*d*tan(c/2 + d*x/2)**6 - 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*t
an(c/2 + d*x/2)**4 - 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c
/2 + d*x/2)**2 - 716636160*d*tan(c/2 + d*x/2) + 71663616*d) + 6101730*log(
tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**5/(71663616*d*tan(c/2 + d*x/2)**6
- 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 - 408
7480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 - 7166361
60*d*tan(c/2 + d*x/2) + 71663616*d) - 22169619*log(tan(c/2 + d*x/2) - 3)*t
an(c/2 + d*x/2)**4/(71663616*d*tan(c/2 + d*x/2)**6 - 716636160*d*tan(c/2 +
 d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 - 4087480320*d*tan(c/2 + d*x
/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 - 716636160*d*tan(c/2 + d*x/2) +
 71663616*d) + 34802460*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**3/(716
63616*d*tan(c/2 + d*x/2)**6 - 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048
*d*tan(c/2 + d*x/2)**4 - 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*t
an(c/2 + d*x/2)**2 - 716636160*d*tan(c/2 + d*x/2) + 71663616*d) - 22169619
*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**2/(71663616*d*tan(c/2 + d*x/2
)**6 - 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4
- 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 -...
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(126) = 252$.

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.96

$$\int \frac{1}{(3-5\sin(c+dx))^4}\,dx$$
$$= \frac{\frac{40\left(\frac{342495\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1066482\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1218910\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{486441\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84915\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-42741\right)}{\frac{270\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{981\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1540\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{981\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{270\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{27\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-27}+22599\,\log\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}\right.}{2654208\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2654208*(40*(342495*sin(d*x + c)/(cos(d*x + c) + 1) - 1066482*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1218910*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 -
486441*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 84915*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 42741)/(270*sin(d*x + c)/(cos(d*x + c) + 1) - 981*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 98
1*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 270*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 27*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 27) + 22599*log(3*sin(d*x
 + c)/(cos(d*x + c) + 1) - 1) - 22599*log(sin(d*x + c)/(cos(d*x + c) + 1)
- 3))/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{1}{(3-5\sin(c+dx))^4}\,dx = -\frac{\frac{40\left(84915\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-486441\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1218910\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-1066482\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+342495\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-42741\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^3}}{2654208\,d}$$

input
```
integrate(1/(3-5*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/2654208*(40*(84915*tan(1/2*d*x + 1/2*c)^5 - 486441*tan(1/2*d*x + 1/2*c)
^4 + 1218910*tan(1/2*d*x + 1/2*c)^3 - 1066482*tan(1/2*d*x + 1/2*c)^2 + 342
495*tan(1/2*d*x + 1/2*c) - 42741)/(3*tan(1/2*d*x + 1/2*c)^2 - 10*tan(1/2*d
*x + 1/2*c) + 3)^3 - 22599*log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) + 22599*lo
g(abs(tan(1/2*d*x + 1/2*c) - 3)))/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int \frac{1}{(3-5\sin(c+dx))^4}\,dx = \frac{279\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{5}{4}\right)}{16384\,d} - \frac{\frac{15725\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{331776}-\frac{270245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{995328}+\frac{3047275\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4478976}-\frac{296245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{497664}+\frac{63425\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{331776}-\frac{79}{331}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{1540\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{27}+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-10\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(1/(5*sin(c + d*x) - 3)^4,x)
```

output
```
(279*atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4))/(16384*d) - ((63425*tan(c/2 +
(d*x)/2))/331776 - (296245*tan(c/2 + (d*x)/2)^2)/497664 + (3047275*tan(c/2
 + (d*x)/2)^3)/4478976 - (270245*tan(c/2 + (d*x)/2)^4)/995328 + (15725*tan
(c/2 + (d*x)/2)^5)/331776 - 7915/331776)/(d*((109*tan(c/2 + (d*x)/2)^2)/3
- 10*tan(c/2 + (d*x)/2) - (1540*tan(c/2 + (d*x)/2)^3)/27 + (109*tan(c/2 +
(d*x)/2)^4)/3 - 10*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.41. $\int \frac{1}{(3-5\sin(c+dx))^4}\,dx$

3.42 $\int \frac{1}{-3+5\sin(c+dx)}\,dx$

3.42.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{1}{-3+5\sin(c+dx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

output
```
1/4*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d-1/4*ln(3*cos(1/2*d*x+1/2
*c)-sin(1/2*d*x+1/2*c))/d
```

3.42.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{-3+5\sin(c+dx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

input
```
Integrate[(-3 + 5*Sin[c + d*x])^(-1),x]
```

output
```
Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]]/(4*d) - Log[3*Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]]/(4*d)
```

3.42.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\sin(c+dx)-3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{5\sin(c+dx)-3}dx$$

$$\Big\downarrow \text{3139}$$

$$\frac{2\int \frac{1}{-3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)-3}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{1081}$$

$$-\frac{6\int \left(\frac{1}{8\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{24\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

input
```
Int[(-3 + 5*Sin[c + d*x])^(-1),x]
```

output
```
(-6*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24))/d
```

3.42.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{4}}{d}$	36
default	$\frac{\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{4}}{d}$	36
parallelrisch	$\frac{-\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-9\right)+\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$	37
norman	$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{4d}+\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$	38
risch	$\frac{\ln\left(-\frac{4}{5}-\frac{3i}{5}+e^{i(dx+c)}\right)}{4d}-\frac{\ln\left(e^{i(dx+c)}+\frac{4}{5}-\frac{3i}{5}\right)}{4d}$	40

input
```
int(1/(-3+5*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*ln(3*tan(1/2*d*x+1/2*c)-1)-1/4*ln(tan(1/2*d*x+1/2*c)-3))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{1}{-3+5\sin(c+dx)}\,dx = -\frac{\log\left(4\cos\left(dx+c\right)-3\sin\left(dx+c\right)+5\right)-\log\left(-4\cos\left(dx+c\right)-3\sin\left(dx+c\right)+5\right)}{8\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(log(4*cos(d*x + c) - 3*sin(d*x + c) + 5) - log(-4*cos(d*x + c) - 3*s
in(d*x + c) + 5))/d
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{1}{-3+5\sin(c+dx)}\,dx = \begin{cases} -\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)}{4d} + \frac{\log\left(3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{5\sin(c)-3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-3+5*sin(d*x+c)),x)
```

output
```
Piecewise((-log(tan(c/2 + d*x/2) - 3)/(4*d) + log(3*tan(c/2 + d*x/2) - 1)/
(4*d), Ne(d, 0)), (x/(5*sin(c) - 3), True))
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{1}{-3+5\sin(c+dx)}\,dx = \frac{\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-1\right) - \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{4\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(log(3*sin(d*x + c)/(cos(d*x + c) + 1) - 1) - log(sin(d*x + c)/(cos(d*
x + c) + 1) - 3))/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{1}{-3+5\sin(c+dx)}\,dx = \frac{\log\left(\left|3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\right|\right)}{4\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) - log(abs(tan(1/2*d*x + 1/2*c) -
 3)))/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.29

$$\int \frac{1}{-3+5\sin(c+dx)}\,dx = \frac{\operatorname{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{5}{4}\right)}{2\,d}$$

input
```
int(1/(5*sin(c + d*x) - 3),x)
```

output
```
atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4)/(2*d)
```

3.43 $\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx$

3.43.1 Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx = \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}$$

output
```
3/64*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d-3/64*ln(3*cos(1/2*d*x+1
/2*c)-sin(1/2*d*x+1/2*c))/d+5/16*cos(d*x+c)/d/(3-5*sin(d*x+c))
```

3.43.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

$$\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx = \frac{9\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+20\left(\frac{\quad}{\cos\left(\frac{1}{2}(c+dx)\right)\ldots}\right.}{192d}$$

input
```
Integrate[(-3 + 5*Sin[c + d*x])^(-2),x]
```

output
```
(9*(Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]] - Log[3*Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]]) + 20*(3/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]) + (3*Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^(-1))*Sin[(c + d*x)/2])/(192*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3143, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(c+dx)-3)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5\sin(c+dx)-3)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{16}\int -\frac{3}{3-5\sin(c+dx)}dx+\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{3}{16}\int \frac{1}{3-5\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{3}{16}\int \frac{1}{3-5\sin(c+dx)}dx$$

$$\Big\downarrow \text{3139}$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{3\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}$$

$$\Big\downarrow \text{1081}$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{9\int \left(\frac{1}{8\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{24\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{9\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}$$

input
```
Int[(-3 + 5*Sin[c + d*x])^(-2),x]
```

output
```
(-9*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24))/(8
*d) + (5*Cos[c + d*x])/(16*d*(3 - 5*Sin[c + d*x]))
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.43.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
default	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
risch	$\frac{3\,\mathrm{e}^{i(dx+c)}-5i}{8d\left(5\,\mathrm{e}^{2i(dx+c)}-5-6i\mathrm{e}^{i(dx+c)}\right)}+\frac{3\ln\left(-\frac{4}{5}-\frac{3i}{5}+\mathrm{e}^{i(dx+c)}\right)}{64d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{4}{5}-\frac{3i}{5}\right)}{64d}$
norman	$\frac{\frac{5}{8d}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}}{3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{64d}+\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64d}$
parallelrisch	$\frac{-45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)\sin(dx+c)+45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3}\right)\sin(dx+c)+27\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)-27\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{192d(-3+5\sin(dx+c))}$

input
```
int(1/(-3+5*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-5/16/(tan(1/2*d*x+1/2*c)-3)-3/64*ln(tan(1/2*d*x+1/2*c)-3)-5/48/(3*ta
n(1/2*d*x+1/2*c)-1)+3/64*ln(3*tan(1/2*d*x+1/2*c)-1))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx =$$
$$-\frac{3\,(5\,\sin\left(dx+c\right)-3)\log\left(4\,\cos\left(dx+c\right)-3\,\sin\left(dx+c\right)+5\right)-3\,(5\,\sin\left(dx+c\right)-3)\log\left(-4\,\cos\left(dx-\right.\right.}{128\,(5\,d\sin\left(dx+c\right)-3\,d)}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/128*(3*(5*sin(d*x + c) - 3)*log(4*cos(d*x + c) - 3*sin(d*x + c) + 5) -
3*(5*sin(d*x + c) - 3)*log(-4*cos(d*x + c) - 3*sin(d*x + c) + 5) + 40*cos(
d*x + c))/(5*d*sin(d*x + c) - 3*d)
```

3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(78) = 156$.

Time = 0.72 (sec) , antiderivative size = 462, normalized size of antiderivative = 5.13

$$\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{x}{\left(-3+5\sin\left(2\operatorname{atan}\left(\frac{1}{3}\right)\right)\right)^2} \\ \frac{x}{\left(-3+5\sin\left(2\operatorname{atan}(3)\right)\right)^2} \\ \frac{x}{(5\sin(c)-3)^2} \\ -\frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d} + \frac{90\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d} - \frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1920d\tan\left(\frac{c}{2}+\right.} \end{cases}$$

input
```
integrate(1/(-3+5*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(-3 + 5*sin(2*atan(1/3)))**2, Eq(c, -d*x + 2*atan(1/3))), (x/
(-3 + 5*sin(2*atan(3)))**2, Eq(c, -d*x + 2*atan(3))), (x/(5*sin(c) - 3)**2
, Eq(d, 0)), (-27*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**2/(576*d*tan
(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d) + 90*log(tan(c/2 + d*x
/2) - 3)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*
x/2) + 576*d) - 27*log(tan(c/2 + d*x/2) - 3)/(576*d*tan(c/2 + d*x/2)**2 -
1920*d*tan(c/2 + d*x/2) + 576*d) + 27*log(3*tan(c/2 + d*x/2) - 1)*tan(c/2
+ d*x/2)**2/(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d)
- 90*log(3*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)*
*2 - 1920*d*tan(c/2 + d*x/2) + 576*d) + 27*log(3*tan(c/2 + d*x/2) - 1)/(57
6*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d) - 200*tan(c/2 +
 d*x/2)/(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d) + 12
0/(576*d*tan(c/2 + d*x/2)**2 - 1920*d*tan(c/2 + d*x/2) + 576*d), True))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{40\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{\frac{10\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-3} + 9\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-1\right) - 9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{192\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/192*(40*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 3)/(10*sin(d*x + c)/(cos(d*
x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3) + 9*log(3*sin(d*x
 + c)/(cos(d*x + c) + 1) - 1) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 3)
)/d
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx = -\frac{\frac{40\left(5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\right)}{3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}-9\log\left(\left|3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\right|\right)}{192\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/192*(40*(5*tan(1/2*d*x + 1/2*c) - 3)/(3*tan(1/2*d*x + 1/2*c)^2 - 10*tan
(1/2*d*x + 1/2*c) + 3) - 9*log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) + 9*log(ab
s(tan(1/2*d*x + 1/2*c) - 3)))/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+5\sin(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{5}{4}\right)}{32\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{72}-\frac{5}{24}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1\right)}$$

input
```
int(1/(5*sin(c + d*x) - 3)^2,x)
```

output
```
(3*atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4))/(32*d) - ((25*tan(c/2 + (d*x)/2)
)/72 - 5/24)/(d*(tan(c/2 + (d*x)/2)^2 - (10*tan(c/2 + (d*x)/2))/3 + 1))
```

3.44 $\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx$

3.44.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx = \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2} + \frac{45\cos(c+dx)}{512d(3-5\sin(c+dx))}$$

output
```
43/2048*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d-43/2048*ln(3*cos(1/2
*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-5/32*cos(d*x+c)/d/(3-5*sin(d*x+c))^2+45/
512*cos(d*x+c)/d/(3-5*sin(d*x+c))
```

3.44.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.59

$$\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx = \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)-43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{\cdots}{(\cos(\frac{1}{2}(c+dx))-\cdots}}{\cdots}$$

input
```
Integrate[(-3 + 5*Sin[c + d*x])^(-3),x]
```

output
```
(43*Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]] - 43*Log[3*Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] - 40/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2])^2 + 60*(
3/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]) + (3*Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])^(-1))*Sin[(c + d*x)/2] + 40/(-3*Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2)/(2048*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3143, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(c+dx)-3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5\sin(c+dx)-3)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{32}\int \frac{5\sin(c+dx)+6}{(3-5\sin(c+dx))^2}dx-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \frac{5\sin(c+dx)+6}{(3-5\sin(c+dx))^2}dx-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(\frac{1}{16}\int -\frac{43}{3-5\sin(c+dx)}dx+\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{43}{16}\int \frac{1}{3-5\sin(c+dx)}dx\right)-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{43}{16}\int \frac{1}{3-5\sin(c+dx)}dx\right)-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{43\int\frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}\right)-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$\Big\downarrow$ 1081

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{129\int\left(\frac{1}{8\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{24\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}\right)-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

$\Big\downarrow$ 2009

$$\frac{1}{32}\left(\frac{45\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{129\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}\right)-\frac{5\cos(c+dx)}{32d(3-5\sin(c+dx))^2}$$

input
```
Int[(-3 + 5*Sin[c + d*x])^(-3),x]
```

output
```
((-129*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24))
/(8*d) + (45*Cos[c + d*x])/(16*d*(3 - 5*Sin[c + d*x])))/32 - (5*Cos[c + d*
x])/(32*d*(3 - 5*Sin[c + d*x])^2)
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.44. $\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.44.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}+\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{2048}-\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+ 43}{d}$
default	$\frac{\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}+\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{2048}-\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+ 43}{d}$
risch	$\frac{-387i\,\mathrm{e}^{2i(dx+c)}+215\,\mathrm{e}^{3i(dx+c)}-325\,\mathrm{e}^{i(dx+c)}+225i}{256\left(5\,\mathrm{e}^{2i(dx+c)}-5-6i\,\mathrm{e}^{i(dx+c)}\right)^{2}d}-\frac{43\ln\left(\mathrm{e}^{i(dx+c)}+\frac{4}{5}-\frac{3i}{5}\right)}{2048d}+\frac{43\ln\left(-\frac{4}{5}-\frac{3i}{5}+\mathrm{e}^{i(dx+c)}\right)}{2048d}$
norman	$\frac{\frac{55}{256d}+\frac{3245\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2304d}+\frac{125\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}-\frac{1225\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{768d}}{\left(3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{2048d}+\frac{43\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{2048d}$
parallelrisch	$\frac{\left(9675\cos\left(2dx+2c\right)+23220\sin\left(dx+c\right)-16641\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3}\right)+\left(-9675\cos\left(2dx+2c\right)-23220\sin\left(dx+c\right)+16641\right)\ln}{18432d\left(-43+25\cos\left(2dx+2c\right)+6\right.}$

input
```
int(1/(-3+5*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(25/128/(tan(1/2*d*x+1/2*c)-3)^2+15/512/(tan(1/2*d*x+1/2*c)-3)-43/2048
*ln(tan(1/2*d*x+1/2*c)-3)-25/1152/(3*tan(1/2*d*x+1/2*c)-1)^2-155/4608/(3*t
an(1/2*d*x+1/2*c)-1)+43/2048*ln(3*tan(1/2*d*x+1/2*c)-1))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx = -\frac{43\left(25\cos\left(dx+c\right)^2+30\sin\left(dx+c\right)-34\right)\log\left(4\cos\left(dx+c\right)-3\sin\left(dx+c\right)+5\right)-43\left(25\cos\left(dx\right.\right.}{4096\left(25\,d\cos\left(\right.\right.}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4096*(43*(25*cos(d*x + c)^2 + 30*sin(d*x + c) - 34)*log(4*cos(d*x + c)
- 3*sin(d*x + c) + 5) - 43*(25*cos(d*x + c)^2 + 30*sin(d*x + c) - 34)*log(
-4*cos(d*x + c) - 3*sin(d*x + c) + 5) - 1800*cos(d*x + c)*sin(d*x + c) + 4
40*cos(d*x + c))/(25*d*cos(d*x + c)^2 + 30*d*sin(d*x + c) - 34*d)
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(102) = 204$.

Time = 1.41 (sec) , antiderivative size = 1224, normalized size of antiderivative = 10.64

$$\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-3+5*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(-3 + 5*sin(2*atan(1/3)))**3, Eq(c, -d*x + 2*atan(1/3))), (x/
(-3 + 5*sin(2*atan(3)))**3, Eq(c, -d*x + 2*atan(3))), (x/(5*sin(c) - 3)**3
, Eq(d, 0)), (-3483*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**4/(165888*
d*tan(c/2 + d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2
+ d*x/2)**2 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 23220*log(tan(c/2 +
 d*x/2) - 3)*tan(c/2 + d*x/2)**3/(165888*d*tan(c/2 + d*x/2)**4 - 1105920*d
*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 - 1105920*d*tan(c/2 +
 d*x/2) + 165888*d) - 45666*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**2/
(165888*d*tan(c/2 + d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*
tan(c/2 + d*x/2)**2 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 23220*log(t
an(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)/(165888*d*tan(c/2 + d*x/2)**4 - 1105
920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 - 1105920*d*tan(
c/2 + d*x/2) + 165888*d) - 3483*log(tan(c/2 + d*x/2) - 3)/(165888*d*tan(c/
2 + d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)
**2 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 3483*log(3*tan(c/2 + d*x/2)
 - 1)*tan(c/2 + d*x/2)**4/(165888*d*tan(c/2 + d*x/2)**4 - 1105920*d*tan(c/
2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 - 1105920*d*tan(c/2 + d*x/2)
 + 165888*d) - 23220*log(3*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**3/(1658
88*d*tan(c/2 + d*x/2)**4 - 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c
/2 + d*x/2)**2 - 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 45666*log(3*t...
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.70

$$\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx = \frac{\frac{40\left(\frac{735\sin(dx+c)}{\cos(dx+c)+1}-\frac{649\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-99\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1}-\frac{118\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-9}+387\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-1\right)-387\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-3\right)}{18432\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/18432*(40*(735*sin(d*x + c)/(cos(d*x + c) + 1) - 649*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 99)/(60*sin(d*
x + c)/(cos(d*x + c) + 1) - 118*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 9) + 387*log(3*sin(d*x + c)/(cos(d*x + c) + 1) - 1) - 387*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 3))/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx = \frac{\frac{40\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+649\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+99\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^2}+387\log\left(\left|3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-387\log\left(\left|\tan\right.\right.}{18432\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/18432*(40*(75*tan(1/2*d*x + 1/2*c)^3 + 649*tan(1/2*d*x + 1/2*c)^2 - 735*
tan(1/2*d*x + 1/2*c) + 99)/(3*tan(1/2*d*x + 1/2*c)^2 - 10*tan(1/2*d*x + 1/
2*c) + 3)^2 + 387*log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) - 387*log(abs(tan(1
/2*d*x + 1/2*c) - 3)))/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-3+5\sin(c+dx))^3}\,dx = \frac{43\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}-\frac{5}{4}\right)}{1024\,d} + \frac{\frac{125\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6912}+\frac{3245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{20736}-\frac{1225\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6912}+\frac{55}{2304}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\frac{20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{118\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{9}-\frac{20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+1\right)}$$

input
```
int(1/(5*sin(c + d*x) - 3)^3,x)
```

output
```
(43*atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4))/(1024*d) + ((3245*tan(c/2 + (d*
x)/2)^2)/20736 - (1225*tan(c/2 + (d*x)/2))/6912 + (125*tan(c/2 + (d*x)/2)^
3)/6912 + 55/2304)/(d*((118*tan(c/2 + (d*x)/2)^2)/9 - (20*tan(c/2 + (d*x)/
2))/3 - (20*tan(c/2 + (d*x)/2)^3)/3 + tan(c/2 + (d*x)/2)^4 + 1))
```

3.45 $\int \frac{1}{(-3+5\sin(c+dx))^4} dx$

3.45.1 Optimal result

Integrand size = 12, antiderivative size = 140

$$\int \frac{1}{(-3+5\sin(c+dx))^4} dx = \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3} - \frac{25\cos(c+dx)}{512d(3-5\sin(c+dx))^2} + \frac{995\cos(c+dx)}{24576d(3-5\sin(c+dx))}$$

output
```
279/32768*ln(cos(1/2*d*x+1/2*c)-3*sin(1/2*d*x+1/2*c))/d-279/32768*ln(3*cos
(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d+5/48*cos(d*x+c)/d/(3-5*sin(d*x+c))^3
-25/512*cos(d*x+c)/d/(3-5*sin(d*x+c))^2+995/24576*cos(d*x+c)/d/(3-5*sin(d*
x+c))
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx$$
$$= \frac{2511\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)-2511\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{}{(\cos\left(\frac{1}{2}(c-\right.}}{}$$

input
```
Integrate[(-3 + 5*Sin[c + d*x])^(-4),x]
```

output
```
(2511*Log[Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2]] - 2511*Log[3*Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - 720/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2])^2 +
 20*(240/(Cos[(c + d*x)/2] - 3*Sin[(c + d*x)/2])^3 + 597/(Cos[(c + d*x)/2]
 - 3*Sin[(c + d*x)/2]) + 80/(3*Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + 19
9/(3*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))*Sin[(c + d*x)/2] + 2320/(-3*Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(294912*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(c+dx)-3)^4}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{(5\sin(c+dx)-3)^4}dx$$
$$\Big\downarrow \ 3143$$
$$\frac{1}{48}\int -\frac{10\sin(c+dx)+9}{(3-5\sin(c+dx))^3}dx+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$
$$\Big\downarrow \ 25$$
$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}-\frac{1}{48}\int \frac{10\sin(c+dx)+9}{(3-5\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}-\frac{1}{48}\int\frac{10\sin(c+dx)+9}{(3-5\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(-\frac{1}{32}\int-\frac{75\sin(c+dx)+154}{(3-5\sin(c+dx))^2}dx-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{48}\left(\frac{1}{32}\int\frac{75\sin(c+dx)+154}{(3-5\sin(c+dx))^2}dx-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{1}{32}\int\frac{75\sin(c+dx)+154}{(3-5\sin(c+dx))^2}dx-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{1}{16}\int-\frac{837}{3-5\sin(c+dx)}dx+\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{837}{16}\int\frac{1}{3-5\sin(c+dx)}dx\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{837}{16}\int\frac{1}{3-5\sin(c+dx)}dx\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{837\int\frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}\right)-\frac{75\cos(c+dx)}{32d(3-5\sin(c+dx))^2}\right)+\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{1081}$$

3.45. $\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{2511\int\left(\frac{1}{8(1-3\tan(\frac{1}{2}(c+dx)))}-\frac{1}{24(3-\tan(\frac{1}{2}(c+dx)))}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}\right)-\frac{75}{32d(3}\right.$$

$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\cos(c+dx)}{48d(3-5\sin(c+dx))^3}+$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\cos(c+dx)}{16d(3-5\sin(c+dx))}-\frac{2511\left(\frac{1}{24}\log\left(3-\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(1-3\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}\right)-\frac{7}{32d(3}\right.$$

input
```
Int[(-3 + 5*Sin[c + d*x])^(-4),x]
```

output
```
(((-2511*(-1/24*Log[1 - 3*Tan[(c + d*x)/2]] + Log[3 - Tan[(c + d*x)/2]]/24
))/(8*d) + (995*Cos[c + d*x])/(16*d*(3 - 5*Sin[c + d*x])))/32 - (75*Cos[c
+ d*x])/(32*d*(3 - 5*Sin[c + d*x])^2))/48 + (5*Cos[c + d*x])/(48*d*(3 - 5*
Sin[c + d*x])^3)
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.45.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{3}}-\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}}{d}$
default	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{3}}-\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}-\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}}{d}$
risch	$\frac{-111042\,e^{3i(dx+c)}-62775ie^{4i(dx+c)}+119310ie^{2i(dx+c)}+20925\,e^{5i(dx+c)}+68625\,e^{i(dx+c)}-24875i}{12288\left(5\,e^{2i(dx+c)}-5-6ie^{i(dx+c)}\right)^{3}d}+\frac{279\ln\left(-\frac{4}{5}-\frac{3i}{5}+\right.}{32768d}$
norman	$\frac{\frac{7915}{12288d}-\frac{15725\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12288d}-\frac{3047275\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{165888d}-\frac{63425\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12288d}+\frac{296245\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{18432d}+\frac{270245\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{36864d}}{\left(3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}$
parallelrisch	$(10169550\cos(2dx+2c)+20678085\sin(dx+c)-2824875\sin(3dx+3c)-12610242)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3}\right)+(-10169550\cos$

input
```
int(1/(-3+5*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-125/768/(tan(1/2*d*x+1/2*c)-3)^3-75/1024/(tan(1/2*d*x+1/2*c)-3)^2-34
5/8192/(tan(1/2*d*x+1/2*c)-3)-279/32768*ln(tan(1/2*d*x+1/2*c)-3)-125/20736
/(3*tan(1/2*d*x+1/2*c)-1)^3-275/27648/(3*tan(1/2*d*x+1/2*c)-1)^2-3505/2211
84/(3*tan(1/2*d*x+1/2*c)-1)+279/32768*ln(3*tan(1/2*d*x+1/2*c)-1))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx = \frac{199000\cos(dx+c)^3 - 837\left(225\cos(dx+c)^2 - 5\left(25\cos(dx+c)^2 - 52\right)\sin(dx+c) - 252\right)\log\left(4\cos\left(\right.\right.}{}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/196608*(199000*cos(d*x + c)^3 - 837*(225*cos(d*x + c)^2 - 5*(25*cos(d*x
+ c)^2 - 52)*sin(d*x + c) - 252)*log(4*cos(d*x + c) - 3*sin(d*x + c) + 5)
+ 837*(225*cos(d*x + c)^2 - 5*(25*cos(d*x + c)^2 - 52)*sin(d*x + c) - 252)
*log(-4*cos(d*x + c) - 3*sin(d*x + c) + 5) + 190800*cos(d*x + c)*sin(d*x +
 c) - 262320*cos(d*x + c))/(225*d*cos(d*x + c)^2 - 5*(25*d*cos(d*x + c)^2
- 52*d)*sin(d*x + c) - 252*d)
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2353 vs. 2(126) = 252.

Time = 3.12 (sec) , antiderivative size = 2353, normalized size of antiderivative = 16.81

$$\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-3+5*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(-3 + 5*sin(2*atan(1/3)))**4, Eq(c, -d*x + 2*atan(1/3))), (x/
(-3 + 5*sin(2*atan(3)))**4, Eq(c, -d*x + 2*atan(3))), (x/(5*sin(c) - 3)**4
, Eq(d, 0)), (-610173*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**6/(71663
616*d*tan(c/2 + d*x/2)**6 - 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d
*tan(c/2 + d*x/2)**4 - 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan
(c/2 + d*x/2)**2 - 716636160*d*tan(c/2 + d*x/2) + 71663616*d) + 6101730*lo
g(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**5/(71663616*d*tan(c/2 + d*x/2)**
6 - 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 - 4
087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 - 71663
6160*d*tan(c/2 + d*x/2) + 71663616*d) - 22169619*log(tan(c/2 + d*x/2) - 3)
*tan(c/2 + d*x/2)**4/(71663616*d*tan(c/2 + d*x/2)**6 - 716636160*d*tan(c/2
 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 - 4087480320*d*tan(c/2 + d
*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 - 716636160*d*tan(c/2 + d*x/2)
 + 71663616*d) + 34802460*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**3/(7
1663616*d*tan(c/2 + d*x/2)**6 - 716636160*d*tan(c/2 + d*x/2)**5 + 26037780
48*d*tan(c/2 + d*x/2)**4 - 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d
*tan(c/2 + d*x/2)**2 - 716636160*d*tan(c/2 + d*x/2) + 71663616*d) - 221696
19*log(tan(c/2 + d*x/2) - 3)*tan(c/2 + d*x/2)**2/(71663616*d*tan(c/2 + d*x
/2)**6 - 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**
4 - 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2...
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(126) = 252$.

Time = 0.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.96

$$\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx = \frac{\frac{40\left(\frac{342495\sin(dx+c)}{\cos(dx+c)+1}-\frac{1066482\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1218910\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{486441\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84915\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-42741\right)}{\frac{270\sin(dx+c)}{\cos(dx+c)+1}-\frac{981\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1540\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{981\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{270\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{27\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-27}+22599\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}\right.}{2654208\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2654208*(40*(342495*sin(d*x + c)/(cos(d*x + c) + 1) - 1066482*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1218910*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 -
486441*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 84915*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 42741)/(270*sin(d*x + c)/(cos(d*x + c) + 1) - 981*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 98
1*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 270*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 27*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 27) + 22599*log(3*sin(d*x
 + c)/(cos(d*x + c) + 1) - 1) - 22599*log(sin(d*x + c)/(cos(d*x + c) + 1)
- 3))/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx = -\frac{\frac{40\left(84915\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-486441\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1218910\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-1066482\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+342495\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-42741\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^3}}{2654208\,d}$$

input
```
integrate(1/(-3+5*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/2654208*(40*(84915*tan(1/2*d*x + 1/2*c)^5 - 486441*tan(1/2*d*x + 1/2*c)
^4 + 1218910*tan(1/2*d*x + 1/2*c)^3 - 1066482*tan(1/2*d*x + 1/2*c)^2 + 342
495*tan(1/2*d*x + 1/2*c) - 42741)/(3*tan(1/2*d*x + 1/2*c)^2 - 10*tan(1/2*d
*x + 1/2*c) + 3)^3 - 22599*log(abs(3*tan(1/2*d*x + 1/2*c) - 1)) + 22599*lo
g(abs(tan(1/2*d*x + 1/2*c) - 3)))/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx = \frac{279\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{5}{4}\right)}{16384\,d} - \frac{\frac{15725\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{331776}-\frac{270245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{995328}+\frac{3047275\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4478976}-\frac{296245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{497664}+\frac{63425\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{331776}-\frac{79}{331}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{1540\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{27}+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-10\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(1/(5*sin(c + d*x) - 3)^4,x)
```

output
```
(279*atanh((3*tan(c/2 + (d*x)/2))/4 - 5/4))/(16384*d) - ((63425*tan(c/2 +
(d*x)/2))/331776 - (296245*tan(c/2 + (d*x)/2)^2)/497664 + (3047275*tan(c/2
 + (d*x)/2)^3)/4478976 - (270245*tan(c/2 + (d*x)/2)^4)/995328 + (15725*tan
(c/2 + (d*x)/2)^5)/331776 - 7915/331776)/(d*((109*tan(c/2 + (d*x)/2)^2)/3
- 10*tan(c/2 + (d*x)/2) - (1540*tan(c/2 + (d*x)/2)^3)/27 + (109*tan(c/2 +
(d*x)/2)^4)/3 - 10*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.45. $\int \frac{1}{(-3+5\sin(c+dx))^4}\,dx$

3.46 $\int \frac{1}{-3-5\sin(c+dx)}\,dx$

3.46.1 Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{1}{-3-5\sin(c+dx)}\,dx = \frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

output
```
1/4*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-1/4*ln(cos(1/2*d*x+1/2*c
)+3*sin(1/2*d*x+1/2*c))/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{-3-5\sin(c+dx)}\,dx = \frac{\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

input
```
Integrate[(-3 - 5*Sin[c + d*x])^(-1),x]
```

output
```
Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(4*d) - Log[Cos[(c + d*x)/2] +
3*Sin[(c + d*x)/2]]/(4*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-5\sin(c+dx)-3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{-5\sin(c+dx)-3}dx$$

$$\Big\downarrow \text{3139}$$

$$\frac{2\int \frac{1}{-3\tan^2\left(\frac{1}{2}(c+dx)\right)-10\tan\left(\frac{1}{2}(c+dx)\right)-3}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{1081}$$

$$-\frac{6\int \left(\frac{1}{8\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{d}$$

input
```
Int[(-3 - 5*Sin[c + d*x])^(-1),x]
```

output
```
(-6*(-1/24*Log[3 + Tan[(c + d*x)/2]] + Log[1 + 3*Tan[(c + d*x)/2]]/24))/d
```

3.46.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{4}-\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}}{d}$	36
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{4}-\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}}{d}$	36
parallelrisch	$\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9\right)-\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d}$	37
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{4d}-\frac{\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d}$	38
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{4}{5}+\frac{3i}{5}\right)}{4d}-\frac{\ln\left(-\frac{4}{5}+\frac{3i}{5}+e^{i(dx+c)}\right)}{4d}$	40

input
```
int(1/(-3-5*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*ln(tan(1/2*d*x+1/2*c)+3)-1/4*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{-3-5\sin(c+dx)}\,dx$$
$$=\frac{\log\left(4\cos\left(dx+c\right)+3\sin\left(dx+c\right)+5\right)-\log\left(-4\cos\left(dx+c\right)+3\sin\left(dx+c\right)+5\right)}{8\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(log(4*cos(d*x + c) + 3*sin(d*x + c) + 5) - log(-4*cos(d*x + c) + 3*si
n(d*x + c) + 5))/d
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{1}{-3-5\sin(c+dx)}\,dx = \begin{cases} \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)}{4d} - \frac{\log\left(3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{-5\sin(c)-3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-3-5*sin(d*x+c)),x)
```

output
```
Piecewise(((log(tan(c/2 + d*x/2) + 3)/(4*d) - log(3*tan(c/2 + d*x/2) + 1)/(
4*d), Ne(d, 0)), (x/(-5*sin(c) - 3), True))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{-3-5\sin(c+dx)}\,dx = -\frac{\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+1\right)-\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{4\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(log(3*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - log(sin(d*x + c)/(cos(d
*x + c) + 1) + 3))/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int \frac{1}{-3-5\sin(c+dx)}\,dx = -\frac{\log\left(\left|3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right|\right)}{4\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) - log(abs(tan(1/2*d*x + 1/2*c)
+ 3)))/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.30

$$\int \frac{1}{-3-5\sin(c+dx)}\,dx = \frac{\operatorname{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{5}{4}\right)}{2\,d}$$

input
```
int(-1/(5*sin(c + d*x) + 3),x)
```

output
```
atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4)/(2*d)
```

3.47 $\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx$

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx = \frac{3\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{5\cos(c+dx)}{16d(3+5\sin(c+dx))}$$

output
```
3/64*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-3/64*ln(cos(1/2*d*x+1/2
*c)+3*sin(1/2*d*x+1/2*c))/d-5/16*cos(d*x+c)/d/(3+5*sin(d*x+c))
```

3.47.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.43

$$\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx = \frac{9\left(\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+20\sin\left(\frac{1}{2}(c+a\right.}{192d}$$

input
```
Integrate[(-3 - 5*Sin[c + d*x])^(-2),x]
```

output
```
(9*(Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 3*
Sin[(c + d*x)/2]]) + 20*Sin[(c + d*x)/2]*((3*Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^(-1) + 3/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])))/(192*d)
```

3.47.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3143, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-5\sin(c+dx)-3)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-5\sin(c+dx)-3)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{16}\int -\frac{3}{5\sin(c+dx)+3}dx-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{3\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{1081}$$

$$-\frac{9\int \left(\frac{1}{8\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{8d}-\frac{5\cos(c+dx)}{16d(5\sin(c+dx)+3)}$$

input
```
Int[(-3 - 5*Sin[c + d*x])^(-2),x]
```

output
```
(-9*(-1/24*Log[3 + Tan[(c + d*x)/2]] + Log[1 + 3*Tan[(c + d*x)/2]]/24))/(8
*d) - (5*Cos[c + d*x])/(16*d*(3 + 5*Sin[c + d*x]))
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.47.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}}{d}$
default	$\frac{-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{64}-\frac{5}{48\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}}{d}$
risch	$-\frac{3\,\mathrm{e}^{i(dx+c)}+5i}{8d\left(5\,\mathrm{e}^{2i(dx+c)}-5+6i\mathrm{e}^{i(dx+c)}\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{4}{5}+\frac{3i}{5}\right)}{64d}-\frac{3\ln\left(-\frac{4}{5}+\frac{3i}{5}+\mathrm{e}^{i(dx+c)}\right)}{64d}$
norman	$\frac{-\frac{5}{8d}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}}{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{64d}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64d}$
parallelrisch	$\frac{45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)\sin(dx+c)-45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{3}\right)\sin(dx+c)-100\sin(dx+c)-60\cos(dx+c)+27\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{192d(3+5\sin(dx+c))}$

input
```
int(1/(-3-5*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-5/16/(tan(1/2*d*x+1/2*c)+3)+3/64*ln(tan(1/2*d*x+1/2*c)+3)-5/48/(3*ta
n(1/2*d*x+1/2*c)+1)-3/64*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx$$
$$= \frac{3\,(5\,\sin\,(dx+c)+3)\log\,(4\,\cos\,(dx+c)+3\,\sin\,(dx+c)+5)-3\,(5\,\sin\,(dx+c)+3)\log\,(-4\,\cos\,(dx+c}{128\,(5\,d\sin\,(dx+c)+3\,d)}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/128*(3*(5*sin(d*x + c) + 3)*log(4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 3
*(5*sin(d*x + c) + 3)*log(-4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 40*cos(d
*x + c))/(5*d*sin(d*x + c) + 3*d)
```

3.47.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(78) = 156$.

Time = 0.72 (sec) , antiderivative size = 468, normalized size of antiderivative = 5.32

$$\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{x}{\left(-3+5\sin\left(2\operatorname{atan}\left(\frac{1}{3}\right)\right)\right)^2} \\ \frac{x}{\left(-3+5\sin\left(2\operatorname{atan}\left(3\right)\right)\right)^2} \\ \frac{x}{\left(-5\sin\left(c\right)-3\right)^2} \\ \frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d} + \frac{90\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+576d} + \frac{27\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)}{576d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1920d\tan\left(\frac{c}{2}+\frac{dx}{2}\right.} \end{cases}$$

input
```
integrate(1/(-3-5*sin(d*x+c))**2,x)
```

output
```
Piecewise((x/(-3 + 5*sin(2*atan(1/3)))**2, Eq(c, -d*x - 2*atan(1/3))), (x/
(-3 + 5*sin(2*atan(3)))**2, Eq(c, -d*x - 2*atan(3))), (x/(-5*sin(c) - 3)**
2, Eq(d, 0)), (27*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**2/(576*d*tan
(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d) + 90*log(tan(c/2 + d*x
/2) + 3)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*
x/2) + 576*d) + 27*log(tan(c/2 + d*x/2) + 3)/(576*d*tan(c/2 + d*x/2)**2 +
1920*d*tan(c/2 + d*x/2) + 576*d) - 27*log(3*tan(c/2 + d*x/2) + 1)*tan(c/2
+ d*x/2)**2/(576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d)
- 90*log(3*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)/(576*d*tan(c/2 + d*x/2)*
*2 + 1920*d*tan(c/2 + d*x/2) + 576*d) - 27*log(3*tan(c/2 + d*x/2) + 1)/(57
6*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d) - 200*tan(c/2 +
 d*x/2)/(576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d) - 12
0/(576*d*tan(c/2 + d*x/2)**2 + 1920*d*tan(c/2 + d*x/2) + 576*d), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx$$
$$= -\frac{\frac{40\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{\frac{10\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+3} + 9\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+1\right) - 9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{192\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/192*(40*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 3)/(10*sin(d*x + c)/(cos(d
*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3) + 9*log(3*sin(d*
x + c)/(cos(d*x + c) + 1) + 1) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 3
))/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx = -\frac{\frac{40\,\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right)}{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}+9\,\log\left(\left|3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right|\right)}{192\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/192*(40*(5*tan(1/2*d*x + 1/2*c) + 3)/(3*tan(1/2*d*x + 1/2*c)^2 + 10*tan
(1/2*d*x + 1/2*c) + 3) + 9*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) - 9*log(ab
s(tan(1/2*d*x + 1/2*c) + 3)))/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-3-5\sin(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{5}{4}\right)}{32\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{72}+\frac{5}{24}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1\right)}$$

input
```
int(1/(5*sin(c + d*x) + 3)^2,x)
```

output
```
(3*atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4))/(32*d) - ((25*tan(c/2 + (d*x)/2)
)/72 + 5/24)/(d*((10*tan(c/2 + (d*x)/2))/3 + tan(c/2 + (d*x)/2)^2 + 1))
```

3.48 $\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx$

3.48.1 Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx = \frac{43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5\cos(c+dx)}{32d(3+5\sin(c+dx))^2} - \frac{45\cos(c+dx)}{512d(3+5\sin(c+dx))}$$

output
```
43/2048*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-43/2048*ln(cos(1/2*d
*x+1/2*c)+3*sin(1/2*d*x+1/2*c))/d+5/32*cos(d*x+c)/d/(3+5*sin(d*x+c))^2-45/
512*cos(d*x+c)/d/(3+5*sin(d*x+c))
```

3.48.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.58

$$\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx = \frac{43\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) - 43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{\cdots}{(3\cos\left(\frac{1}{2}(c+dx)\right)\cdots}}{\cdots}$$

input
```
Integrate[(-3 - 5*Sin[c + d*x])^(-3),x]
```

output
```
(43*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 43*Log[Cos[(c + d*x)/2] +
 3*Sin[(c + d*x)/2]] - 40/(3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 40/(
Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])^2 + 60*Sin[(c + d*x)/2]*((3*Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^(-1) + 3/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)
/2])))/(2048*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3143, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-5\sin(c+dx)-3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-5\sin(c+dx)-3)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{32}\int \frac{6-5\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx+\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \frac{6-5\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx+\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(\frac{1}{16}\int -\frac{43}{5\sin(c+dx)+3}dx-\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(-\frac{43}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(-\frac{43}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{32}\left(-\frac{43\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{32}\left(-\frac{129\int \left(\frac{1}{8\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}+\frac{1}{32}\left(-\frac{129\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{8d}-\frac{45\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)$$

input
```
Int[(-3 - 5*Sin[c + d*x])^(-3),x]
```

output
```
(5*Cos[c + d*x])/(32*d*(3 + 5*Sin[c + d*x])^2) + ((-129*(-1/24*Log[3 + Tan
[(c + d*x)/2]] + Log[1 + 3*Tan[(c + d*x)/2]]/24))/(8*d) - (45*Cos[c + d*x]
)/(16*d*(3 + 5*Sin[c + d*x])))/32
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.48.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}+\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{2048}+\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$
default	$\frac{-\frac{25}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}+\frac{15}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{2048}+\frac{25}{1152\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{155}{4608\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$
risch	$-\frac{387i e^{2i(dx+c)}+215\,e^{3i(dx+c)}-325\,e^{i(dx+c)}-225i}{256\left(5\,e^{2i(dx+c)}-5+6i e^{i(dx+c)}\right)^{2} d}+\frac{43\ln\left(e^{i(dx+c)}+\frac{4}{5}+\frac{3i}{5}\right)}{2048d}-\frac{43\ln\left(-\frac{4}{5}+\frac{3i}{5}+e^{i(dx+c)}\right)}{2048d}$
norman	$\frac{-\frac{55}{256d}-\frac{3245\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2304d}-\frac{1225\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{768d}+\frac{125\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}}{\left(3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{2048d}-\frac{43\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2048d}$
parallelrisch	$\frac{(-9675\cos(2dx+2c)+23220\sin(dx+c)+16641)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{3}\right)+(9675\cos(2dx+2c)-23220\sin(dx+c)-16641)\ln}{18432d(-43+25\cos(2dx+2c)-6}$

input
```
int(1/(-3-5*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-25/128/(tan(1/2*d*x+1/2*c)+3)^2+15/512/(tan(1/2*d*x+1/2*c)+3)+43/204
8*ln(tan(1/2*d*x+1/2*c)+3)+25/1152/(3*tan(1/2*d*x+1/2*c)+1)^2-155/4608/(3*
tan(1/2*d*x+1/2*c)+1)-43/2048*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx = \frac{43\left(25\cos(dx+c)^2-30\sin(dx+c)-34\right)\log(4\cos(dx+c)+3\sin(dx+c)+5)-43\left(25\cos(dx+c\right.}{4096\left(25\,d\cos(dx\right.}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4096*(43*(25*cos(d*x + c)^2 - 30*sin(d*x + c) - 34)*log(4*cos(d*x + c) +
 3*sin(d*x + c) + 5) - 43*(25*cos(d*x + c)^2 - 30*sin(d*x + c) - 34)*log(-
4*cos(d*x + c) + 3*sin(d*x + c) + 5) + 1800*cos(d*x + c)*sin(d*x + c) + 44
0*cos(d*x + c))/(25*d*cos(d*x + c)^2 - 30*d*sin(d*x + c) - 34*d)
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. 2(102) = 204.

Time = 1.42 (sec) , antiderivative size = 1229, normalized size of antiderivative = 10.88

$$\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-3-5*sin(d*x+c))**3,x)
```

output
```
Piecewise((x/(-3 + 5*sin(2*atan(1/3)))**3, Eq(c, -d*x - 2*atan(1/3))), (x/
(-3 + 5*sin(2*atan(3)))**3, Eq(c, -d*x - 2*atan(3))), (x/(-5*sin(c) - 3)**
3, Eq(d, 0)), (3483*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**4/(165888*
d*tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2
+ d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 23220*log(tan(c/2 +
 d*x/2) + 3)*tan(c/2 + d*x/2)**3/(165888*d*tan(c/2 + d*x/2)**4 + 1105920*d
*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 + 1105920*d*tan(c/2 +
 d*x/2) + 165888*d) + 45666*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**2/
(165888*d*tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*
tan(c/2 + d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) + 23220*log(t
an(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)/(165888*d*tan(c/2 + d*x/2)**4 + 1105
920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 + 1105920*d*tan(
c/2 + d*x/2) + 165888*d) + 3483*log(tan(c/2 + d*x/2) + 3)/(165888*d*tan(c/
2 + d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)
**2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 3483*log(3*tan(c/2 + d*x/2)
 + 1)*tan(c/2 + d*x/2)**4/(165888*d*tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/
2 + d*x/2)**3 + 2174976*d*tan(c/2 + d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2)
 + 165888*d) - 23220*log(3*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**3/(1658
88*d*tan(c/2 + d*x/2)**4 + 1105920*d*tan(c/2 + d*x/2)**3 + 2174976*d*tan(c
/2 + d*x/2)**2 + 1105920*d*tan(c/2 + d*x/2) + 165888*d) - 45666*log(3*t...
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.73

$$\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{40\left(\frac{735\sin(dx+c)}{\cos(dx+c)+1}+\frac{649\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+99\right)}{\frac{60\sin(dx+c)}{\cos(dx+c)+1}+\frac{118\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+9}+387\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+1\right)-387\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+3}{18432\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/18432*(40*(735*sin(d*x + c)/(cos(d*x + c) + 1) + 649*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 99)/(60*sin(d
*x + c)/(cos(d*x + c) + 1) + 118*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 9) + 387*log(3*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - 387*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 3))/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{40\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-649\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-99\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^2}-387\log\left(\left|3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)+387\log\left(\left|\tan\right.\right.}{18432\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/18432*(40*(75*tan(1/2*d*x + 1/2*c)^3 - 649*tan(1/2*d*x + 1/2*c)^2 - 735*
tan(1/2*d*x + 1/2*c) - 99)/(3*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1/
2*c) + 3)^2 - 387*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) + 387*log(abs(tan(1
/2*d*x + 1/2*c) + 3)))/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 7.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{1}{(-3-5\sin(c+dx))^3}\,dx$$
$$= \frac{43\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{5}{4}\right)}{1024\,d} - \frac{-\frac{125\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6912}+\frac{3245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{20736}+\frac{1225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6912}+\frac{55}{2304}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{118\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{9}+\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1\right)}$$

input
```
int(-1/(5*sin(c + d*x) + 3)^3,x)
```

output
```
(43*atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4))/(1024*d) - ((1225*tan(c/2 + (d*
x)/2))/6912 + (3245*tan(c/2 + (d*x)/2)^2)/20736 - (125*tan(c/2 + (d*x)/2)^
3)/6912 + 55/2304)/(d*((20*tan(c/2 + (d*x)/2))/3 + (118*tan(c/2 + (d*x)/2)
^2)/9 + (20*tan(c/2 + (d*x)/2)^3)/3 + tan(c/2 + (d*x)/2)^4 + 1))
```

3.49 $\int \frac{1}{(-3-5\sin(c+dx))^4} \, dx$

3.49.1 Optimal result

Integrand size = 12, antiderivative size = 138

$$\int \frac{1}{(-3-5\sin(c+dx))^4} \, dx = \frac{279\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{5\cos(c+dx)}{48d(3+5\sin(c+dx))^3} + \frac{25\cos(c+dx)}{512d(3+5\sin(c+dx))^2} - \frac{995\cos(c+dx)}{24576d(3+5\sin(c+dx))}$$

output
```
279/32768*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-279/32768*ln(cos(1
/2*d*x+1/2*c)+3*sin(1/2*d*x+1/2*c))/d-5/48*cos(d*x+c)/d/(3+5*sin(d*x+c))^3
+25/512*cos(d*x+c)/d/(3+5*sin(d*x+c))^2-995/24576*cos(d*x+c)/d/(3+5*sin(d*
x+c))
```

3.49.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.70

$$\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx$$
$$= \frac{2511\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2511\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{\cdots}{(3\cos(\frac{1}{2}(\cdots}}{\cdots}$$

input
```
Integrate[(-3 - 5*Sin[c + d*x])^(-4),x]
```

output
```
(2511*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2511*Log[Cos[(c + d*x)/
2] + 3*Sin[(c + d*x)/2]] - 2320/(3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
+ 720/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])^2 + 20*Sin[(c + d*x)/2]*(80/
(3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 199/(3*Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]) + 240/(Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2])^3 + 597/(Cos[(
c + d*x)/2] + 3*Sin[(c + d*x)/2])))/(294912*d)
```

3.49.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-5\sin(c+dx)-3)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(-5\sin(c+dx)-3)^4}dx$$
$$\Big\downarrow \text{3143}$$
$$\frac{1}{48}\int -\frac{9-10\sin(c+dx)}{(5\sin(c+dx)+3)^3}dx-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{48}\int \frac{9-10\sin(c+dx)}{(5\sin(c+dx)+3)^3}dx-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{48}\int \frac{9-10\sin(c+dx)}{(5\sin(c+dx)+3)^3}dx-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}-\frac{1}{32}\int -\frac{154-75\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75\sin(c+dx)}{(5\sin(c+dx)+3)^2}dx+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{1}{16}\int -\frac{837}{5\sin(c+dx)+3}dx-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{837}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{837}{16}\int \frac{1}{5\sin(c+dx)+3}dx-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}\right)-\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{837\int \frac{1}{3\tan^2\left(\frac{1}{2}(c+dx)\right)+10\tan\left(\frac{1}{2}(c+dx)\right)+3}d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{75\cos(c+d}{32d(5\sin(c+dx}\right.$$

$$\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{1081}$$

3.49. $\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx$

$$\frac{1}{48}\left(\frac{1}{32}\left(-\frac{2511\int\left(\frac{1}{8\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\tan\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{995\cos(c+dx)}{16d(5\sin(c+dx)+3)}\right)+\frac{}{32d(}\right.$$
$$\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{48}\left(\frac{75\cos(c+dx)}{32d(5\sin(c+dx)+3)^2}+\frac{1}{32}\left(-\frac{2511\left(\frac{1}{24}\log\left(3\tan\left(\frac{1}{2}(c+dx)\right)+1\right)-\frac{1}{24}\log\left(\tan\left(\frac{1}{2}(c+dx)\right)+3\right)\right)}{8d}-\frac{9}{16d(}\right.\right.$$
$$\frac{5\cos(c+dx)}{48d(5\sin(c+dx)+3)^3}$$

input
```
Int[(-3 - 5*Sin[c + d*x])^(-4),x]
```

output
```
(-5*Cos[c + d*x])/(48*d*(3 + 5*Sin[c + d*x])^3) + ((75*Cos[c + d*x])/(32*d
*(3 + 5*Sin[c + d*x])^2) + ((-2511*(-1/24*Log[3 + Tan[(c + d*x)/2]] + Log[
1 + 3*Tan[(c + d*x)/2]]/24))/(8*d) - (995*Cos[c + d*x])/(16*d*(3 + 5*Sin[c
 + d*x])))/32)/48
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.49.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}+\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{d}$
default	$\frac{-\frac{125}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}+\frac{75}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{2}}-\frac{345}{8192\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{32768}-\frac{125}{20736\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{d}$
risch	$-\frac{-111042\,e^{3i(dx+c)}+62775ie^{4i(dx+c)}-119310ie^{2i(dx+c)}+20925\,e^{5i(dx+c)}+68625\,e^{i(dx+c)}+24875i}{12288\left(5\,e^{2i(dx+c)}-5+6ie^{i(dx+c)}\right)^{3}d}+\frac{279\ln\left(e^{i(dx+c)}\right)}{32768}$
norman	$\frac{-\frac{7915}{12288d}-\frac{63425\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12288d}-\frac{3047275\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{165888d}-\frac{15725\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12288d}-\frac{296245\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{18432d}-\frac{270245\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{36864d}}{\left(3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)^{3}}$
parallelrisch	$\left(-10169550\cos(2dx+2c)+20678085\sin(dx+c)-2824875\sin(3dx+3c)+12610242\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{3}\right)+(10169550\cos$

input
```
int(1/(-3-5*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-125/768/(tan(1/2*d*x+1/2*c)+3)^3+75/1024/(tan(1/2*d*x+1/2*c)+3)^2-34
5/8192/(tan(1/2*d*x+1/2*c)+3)+279/32768*ln(tan(1/2*d*x+1/2*c)+3)-125/20736
/(3*tan(1/2*d*x+1/2*c)+1)^3+275/27648/(3*tan(1/2*d*x+1/2*c)+1)^2-3505/2211
84/(3*tan(1/2*d*x+1/2*c)+1)-279/32768*ln(3*tan(1/2*d*x+1/2*c)+1))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx =$$
$$-\frac{199000\cos(dx+c)^3 - 837\left(225\cos(dx+c)^2 + 5\left(25\cos(dx+c)^2 - 52\right)\sin(dx+c) - 252\right)\log(4\,\mathrm{co}}{}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/196608*(199000*cos(d*x + c)^3 - 837*(225*cos(d*x + c)^2 + 5*(25*cos(d*x
 + c)^2 - 52)*sin(d*x + c) - 252)*log(4*cos(d*x + c) + 3*sin(d*x + c) + 5)
 + 837*(225*cos(d*x + c)^2 + 5*(25*cos(d*x + c)^2 - 52)*sin(d*x + c) - 252
)*log(-4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 190800*cos(d*x + c)*sin(d*x
+ c) - 262320*cos(d*x + c))/(225*d*cos(d*x + c)^2 + 5*(25*d*cos(d*x + c)^2
 - 52*d)*sin(d*x + c) - 252*d)
```

3.49.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2358 vs. 2(126) = 252.

Time = 3.13 (sec) , antiderivative size = 2358, normalized size of antiderivative = 17.09

$$\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-3-5*sin(d*x+c))**4,x)
```

output
```
Piecewise((x/(-3 + 5*sin(2*atan(1/3)))**4, Eq(c, -d*x - 2*atan(1/3))), (x/
(-3 + 5*sin(2*atan(3)))**4, Eq(c, -d*x - 2*atan(3))), (x/(-5*sin(c) - 3)**
4, Eq(d, 0)), (610173*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**6/(71663
616*d*tan(c/2 + d*x/2)**6 + 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d
*tan(c/2 + d*x/2)**4 + 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan
(c/2 + d*x/2)**2 + 716636160*d*tan(c/2 + d*x/2) + 71663616*d) + 6101730*lo
g(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**5/(71663616*d*tan(c/2 + d*x/2)**
6 + 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 + 4
087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 + 71663
6160*d*tan(c/2 + d*x/2) + 71663616*d) + 22169619*log(tan(c/2 + d*x/2) + 3)
*tan(c/2 + d*x/2)**4/(71663616*d*tan(c/2 + d*x/2)**6 + 716636160*d*tan(c/2
 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**4 + 4087480320*d*tan(c/2 + d
*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2 + 716636160*d*tan(c/2 + d*x/2)
 + 71663616*d) + 34802460*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**3/(7
1663616*d*tan(c/2 + d*x/2)**6 + 716636160*d*tan(c/2 + d*x/2)**5 + 26037780
48*d*tan(c/2 + d*x/2)**4 + 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d
*tan(c/2 + d*x/2)**2 + 716636160*d*tan(c/2 + d*x/2) + 71663616*d) + 221696
19*log(tan(c/2 + d*x/2) + 3)*tan(c/2 + d*x/2)**2/(71663616*d*tan(c/2 + d*x
/2)**6 + 716636160*d*tan(c/2 + d*x/2)**5 + 2603778048*d*tan(c/2 + d*x/2)**
4 + 4087480320*d*tan(c/2 + d*x/2)**3 + 2603778048*d*tan(c/2 + d*x/2)**2...
```

3.49.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(124) = 248$.

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.99

$$\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx = -\frac{\frac{40\left(\frac{342495\sin(dx+c)}{\cos(dx+c)+1}+\frac{1066482\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1218910\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{486441\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84915\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+42741\right)}{\frac{270\sin(dx+c)}{\cos(dx+c)+1}+\frac{981\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1540\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{981\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{270\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{27\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+27}+22599\log\left(\frac{3\sin(dx+c}{\cos(dx+c)+}\right.}{2654208\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/2654208*(40*(342495*sin(d*x + c)/(cos(d*x + c) + 1) + 1066482*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 1218910*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 486441*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 84915*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5 + 42741)/(270*sin(d*x + c)/(cos(d*x + c) + 1) + 981*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9
81*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 270*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 27*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 27) + 22599*log(3*sin(d*
x + c)/(cos(d*x + c) + 1) + 1) - 22599*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 3))/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx = -\frac{\frac{40\left(84915\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+486441\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1218910\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1066482\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+342495\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+42741\right)}{\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\right)^3}}{2654208\,d}$$

input
```
integrate(1/(-3-5*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/2654208*(40*(84915*tan(1/2*d*x + 1/2*c)^5 + 486441*tan(1/2*d*x + 1/2*c)
^4 + 1218910*tan(1/2*d*x + 1/2*c)^3 + 1066482*tan(1/2*d*x + 1/2*c)^2 + 342
495*tan(1/2*d*x + 1/2*c) + 42741)/(3*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d
*x + 1/2*c) + 3)^3 + 22599*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) - 22599*lo
g(abs(tan(1/2*d*x + 1/2*c) + 3)))/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.22

$$\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx = \frac{279\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{5}{4}\right)}{16384\,d} - \frac{\frac{15725\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{331776}+\frac{270245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{995328}+\frac{3047275\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4478976}+\frac{296245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{497664}+\frac{63425\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{331776}+\frac{79}{331}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{109\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+\frac{1540\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{27}+\frac{109\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+10\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(1/(5*sin(c + d*x) + 3)^4,x)
```

output
```
(279*atanh((3*tan(c/2 + (d*x)/2))/4 + 5/4))/(16384*d) - ((63425*tan(c/2 +
(d*x)/2))/331776 + (296245*tan(c/2 + (d*x)/2)^2)/497664 + (3047275*tan(c/2
 + (d*x)/2)^3)/4478976 + (270245*tan(c/2 + (d*x)/2)^4)/995328 + (15725*tan
(c/2 + (d*x)/2)^5)/331776 + 7915/331776)/(d*(10*tan(c/2 + (d*x)/2) + (109*
tan(c/2 + (d*x)/2)^2)/3 + (1540*tan(c/2 + (d*x)/2)^3)/27 + (109*tan(c/2 +
(d*x)/2)^4)/3 + 10*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.49. $\int \frac{1}{(-3-5\sin(c+dx))^4}\,dx$

3.50 $\int (a+b\sin(c+dx))^{7/2}\,dx$

3.50.1 Optimal result

Integrand size = 14, antiderivative size = 256

$$\begin{aligned}\int (a+b\sin(c+dx))^{7/2}\,dx &= -\frac{2b(71a^2+25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105d}\\ &\quad-\frac{24ab\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35d}-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}\\ &\quad+\frac{32a(11a^2+13b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{105d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}\\ &\quad-\frac{2(71a^4-46a^2b^2-25b^4)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{105d\sqrt{a+b\sin(c+dx)}}\end{aligned}$$

output
```
-24/35*a*b*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/d-2/7*b*cos(d*x+c)*(a+b*sin(d
*x+c))^(5/2)/d-2/105*b*(71*a^2+25*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/d
-32/105*a*(11*a^2+13*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))
*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+2/105*(71*a^4-46*
a^2*b^2-25*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.86

$$\int (a + b\sin(c+dx))^{7/2}\,dx = \frac{-64a(11a^3 + 11a^2b + 13ab^2 + 13b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + 4(71a^4 - 46a^2 \cdots}{\cdots}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(7/2),x]
```

output
```
(-64*a*(11*a^3 + 11*a^2*b + 13*a*b^2 + 13*b^3)*EllipticE[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 4*(71*a^4 - 46*a
^2*b^2 - 25*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)] - b*Cos[c + d*x]*(488*a^3 + 262*a*b^2 - 162*a*b^
2*Cos[2*(c + d*x)] + b*(752*a^2 + 145*b^2)*Sin[c + d*x] - 15*b^3*Sin[3*(c
+ d*x)]))/(210*d*Sqrt[a + b*Sin[c + d*x]])
```

3.50.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {3042, 3135, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\sin(c+dx))^{7/2}dx$$

$$\Big\downarrow \; 3135$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\sin(c+dx))^{3/2}\left(7a^2+12b\sin(c+dx)a+5b^2\right)dx-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int (a+b\sin(c+dx))^{3/2}\left(7a^2+12b\sin(c+dx)a+5b^2\right)dx-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int (a+b\sin(c+dx))^{3/2}\left(7a^2+12b\sin(c+dx)a+5b^2\right)dx-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sin(c+dx)}\left(a\left(35a^2+61b^2\right)+b\left(71a^2+25b^2\right)\sin(c+dx)\right)dx-\frac{24ab\cos(c+dx)(a+b\sin(c+a}{5d}\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{a+b\sin(c+dx)}\left(a\left(35a^2+61b^2\right)+b\left(71a^2+25b^2\right)\sin(c+dx)\right)dx-\frac{24ab\cos(c+dx)(a+b\sin(c+dx}{5d}\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{a+b\sin(c+dx)}\left(a\left(35a^2+61b^2\right)+b\left(71a^2+25b^2\right)\sin(c+dx)\right)dx-\frac{24ab\cos(c+dx)(a+b\sin(c+dx}{5d}\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{105a^4+254b^2a^2+16b\left(11a^2+13b^2\right)\sin(c+dx)a+25b^4}{2\sqrt{a+b\sin(c+dx)}}dx-\frac{2b\left(71a^2+25b^2\right)\cos(c+dx)\sqrt{a+b\sin}}{3d}\right.\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{105a^4+254b^2a^2+16b\left(11a^2+13b^2\right)\sin(c+dx)a+25b^4}{\sqrt{a+b\sin(c+dx)}}dx-\frac{2b\left(71a^2+25b^2\right)\cos(c+dx)\sqrt{a+b\sin}}{3d}\right.\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

3.50. $\int (a+b\sin(c+dx))^{7/2}\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{105a^4+254b^2a^2+16b(11a^2+13b^2)\sin(c+dx)a+25b^4}{\sqrt{a+b\sin(c+dx)}}dx-\frac{2b(71a^2+25b^2)\cos(c+dx)\sqrt{a+b\sin}}{3d}\right.\right.$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3231$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(16a(11a^2+13b^2)\int\sqrt{a+b\sin(c+dx)}dx-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{2b(7}{}\right.\right.$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(16a(11a^2+13b^2)\int\sqrt{a+b\sin(c+dx)}dx-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{2b(7}{}\right.\right.$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{16a(11a^2+13b^2)\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\sin(}}\right.\right.\right.$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{16a(11a^2+13b^2)\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\sin(}}\right.\right.\right.$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{32a(11a^2+13b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\sin}}\right.\right.\right.$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3142$$

3.50. $\int (a+b\sin(c+dx))^{7/2}\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{32a(11a^2+13b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(71a^4-46a^2b^2-25b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx}}\right.\right.\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{32a(11a^2+13b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(71a^4-46a^2b^2-25b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx}}\right.\right.\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3140

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{32a(11a^2+13b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2(71a^4-46a^2b^2-25b^4)\sqrt{\frac{a+b\sin(c+dx}{a+b}}}{d\sqrt{a+b\sin}}\right.\right.\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7d}$$

input
```
Int[(a + b*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(7*d) + ((-24*a*b*Cos[c + d
*x]*(a + b*Sin[c + d*x])^(3/2))/(5*d) + ((-2*b*(71*a^2 + 25*b^2)*Cos[c + d
*x]*Sqrt[a + b*Sin[c + d*x]])/(3*d) + ((32*a*(11*a^2 + 13*b^2)*EllipticE[(
c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) - (2*(71*a^4 - 46*a^2*b^2 - 25*b^4)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqr
t[a + b*Sin[c + d*x]]))/3)/5)/7
```

3.50. $\int (a+b\sin(c+dx))^{7/2}\,dx$

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.50.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1039 vs. $2(298) = 596$.

Time = 1.19 (sec) , antiderivative size = 1040, normalized size of antiderivative = 4.06

method	result	size
default	Expression too large to display	1040

input
```
int((a+b*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*b^5*sin(d*x+c)^5+105*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+71*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+78*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^3*b^2-46*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^2*b^3-183*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-25*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^5-176*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^5-32*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^3*b^2+208*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(...
```

3.50.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.92

$$\int (a+b\sin(c+dx))^{7/2}\,dx =$$

$$-\frac{\sqrt{2}(37\,a^4-346\,a^2b^2-75\,b^4)\sqrt{i\,b}\,\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)-2}{3\,b}\right.}{}$$

input
```
integrate((a+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/315*(sqrt(2)*(37*a^4 - 346*a^2*b^2 - 75*b^4)*sqrt(I*b)*weierstrassPInve
rse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(37*a^4 - 346*a^2*b^
2 - 75*b^4)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) +
2*I*a)/b) + 48*sqrt(2)*(11*I*a^3*b + 13*I*a*b^3)*sqrt(I*b)*weierstrassZeta
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPIn
verse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*
cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 48*sqrt(2)*(-11*I*a^3*b -
 13*I*a*b^3)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-
8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c)
 + 2*I*a)/b)) - 6*(15*b^4*cos(d*x + c)^3 - 66*a*b^3*cos(d*x + c)*sin(d*x +
 c) - 2*(61*a^2*b^2 + 20*b^4)*cos(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b*d
)
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.50.7 Maxima [F]

$$\int (a+b\sin(c+dx))^{7/2}\,dx=\int (b\sin{(dx+c)}+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(7/2), x)
```

3.50.8 Giac [F]

$$\int (a+b\sin(c+dx))^{7/2}\,dx=\int (b\sin{(dx+c)}+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(7/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^{7/2}\,dx=\int (a+b\,\sin{(c+d\,x)})^{7/2}\,dx$$

input
```
int((a + b*sin(c + d*x))^(7/2),x)
```

output
```
int((a + b*sin(c + d*x))^(7/2), x)
```

3.51 $\int (a+b\sin(c+dx))^{5/2}\,dx$

3.51.1 Optimal result

Integrand size = 14, antiderivative size = 207

$$\begin{aligned}\int (a+b\sin(c+dx))^{5/2}\,dx &= -\frac{16ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15d} \\ &- \frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d} \\ &+ \frac{2(23a^2+9b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{15d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} \\ &- \frac{16a(a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{15d\sqrt{a+b\sin(c+dx)}}\end{aligned}$$

output
```
-2/5*b*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/d-16/15*a*b*cos(d*x+c)*(a+b*sin(d
*x+c))^(1/2)/d-2/15*(23*a^2+9*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+16/15*a*
(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c
))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int (a+b\sin(c+dx))^{5/2}\,dx = \frac{-2(23a^3+23a^2b+9ab^2+9b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+16a(a^2-b^2)\,\text{Ellipt}}{1}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 16*a*(a^2 - b^2)*Elli
pticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)] + b*Cos[c + d*x]*(-22*a^2 - 3*b^2 + 3*b^2*Cos[2*(c + d*x)] - 28*a*b*S
in[c + d*x]))/(15*d*Sqrt[a + b*Sin[c + d*x]])
```

3.51.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 3135, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sin(c+dx)}\left(5a^2+8b\sin(c+dx)a+3b^2\right)dx-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\sin(c+dx)}\left(5a^2+8b\sin(c+dx)a+3b^2\right)dx-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \sqrt{a+b\sin(c+dx)}\left(5a^2+8b\sin(c+dx)a+3b^2\right)dx-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin(c+dx)}{2\sqrt{a+b\sin(c+dx)}}dx-\frac{16ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}\right)-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx-\frac{16ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}\right)-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx-\frac{16ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}\right)-\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3231$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+9b^2\right)\int \sqrt{a+b\sin(c+dx)}dx-8a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{16ab\cos(c+dx)\sqrt{a+b}}{3d}\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+9b^2\right)\int \sqrt{a+b\sin(c+dx)}dx-8a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{16ab\cos(c+dx)\sqrt{a+b}}{3d}\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{16ab}{}\right.$$
$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

3.51. $\int (a+b\sin(c+dx))^{5/2}\,dx$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{16ab\ldots$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{16a\ldots$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3142

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}d\ldots}{\sqrt{a+b\sin(c+dx)}}\ldots$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}d\ldots}{\sqrt{a+b\sin(c+dx)}}\ldots$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3140

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+\ldots\right.}{d\sqrt{a+b\sin(c+dx)}}\ldots$$

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5d}$$

input

```
Int[(a + b*Sin[c + d*x])^(5/2),x]
```

3.51. $\int(a+b\sin(c+dx))^{5/2}\,dx$

output
```
(-2*b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(5*d) + ((-16*a*b*Cos[c + d
*x]*Sqrt[a + b*Sin[c + d*x]])/(3*d) + ((2*(23*a^2 + 9*b^2)*EllipticE[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b*Sin
[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]
]))/3)/5
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.51.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(253) = 506$.

Time = 0.80 (sec) , antiderivative size = 890, normalized size of antiderivative = 4.30

method	result
default	$\frac{2a^4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,F\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)+\frac{16\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+}{}}}{15}}{}$

input
```
int((a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(15*a^4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b-6*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-8*a*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3
-9*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*b^4-23*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^4+14*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2+9*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4+3*b^4*s
in(d*x+c)^4+14*a*b^3*sin(d*x+c)^3+11*a^2*b^2*sin(d*x+c)^2-3*b^4*sin(d*x+c)
^2-14*a*b^3*sin(d*x+c)-11*a^2*b^2)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/...
```

3.51.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.16

$$\int (a + b\sin(c + dx))^{5/2}\, dx =$$

$$-\frac{\sqrt{2}(a^3 - 33\,ab^2)\sqrt{i\,b}\,\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3}, \frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3\,b}\right) + \sqrt{2}(a^3}{}$$

input
```
integrate((a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/45*(sqrt(2)*(a^3 - 33*a*b^2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(a^3 - 33*a*b^2)*sqrt(-I*b)*weierstr
assPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*sqrt(2)*(23*I*a^2
*b + 9*I*b^3)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) -
 2*I*a)/b)) + 3*sqrt(2)*(-23*I*a^2*b - 9*I*b^3)*sqrt(-I*b)*weierstrassZeta
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPI
nverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 6*(3*b^3*cos(d*x + c)*s
in(d*x + c) + 11*a*b^2*cos(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b*d)
```

3.51.6 Sympy [F]

$$\int (a+b\sin(c+dx))^{5/2}\,dx=\int (a+b\sin{(c+dx)})^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sin(c + d*x))**(5/2), x)
```

3.51.7 Maxima [F]

$$\int (a+b\sin(c+dx))^{5/2}\,dx=\int (b\sin{(dx+c)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(5/2), x)
```

3.51.8 Giac [F]

$$\int (a+b\sin(c+dx))^{5/2}\,dx = \int (b\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(5/2), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^{5/2}\,dx = \int (a+b\,\sin(c+d\,x))^{5/2}\,dx$$

input
```
int((a + b*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*sin(c + d*x))^(5/2), x)
```

3.52 $\int (a+b\sin(c+dx))^{3/2}\,dx$

3.52.1 Optimal result

Integrand size = 14, antiderivative size = 167

$$\int (a+b\sin(c+dx))^{3/2}\,dx = -\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d} + \frac{8aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} - \frac{2(a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{3d\sqrt{a+b\sin(c+dx)}}$$

output
```
-2/3*b*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/d-8/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2
^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^
(1/2)+2/3*(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b
*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int (a+b\sin(c+dx))^{3/2}\,dx = \frac{-2b\cos(c+dx)(a+b\sin(c+dx)) - 8a(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + 2(a}{3d\sqrt{a+b\sin(c+dx)}}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*b*Cos[c + d*x]*(a + b*Sin[c + d*x]) - 8*a*(a + b)*EllipticE[(-2*c + Pi
 - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 2*(a^2 -
b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(3*d*Sqrt[a + b*Sin[c + d*x]])
```

3.52.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3135, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{3}\int \frac{3a^2+4b\sin(c+dx)a+b^2}{2\sqrt{a+b\sin(c+dx)}}dx - \frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+4b\sin(c+dx)a+b^2}{\sqrt{a+b\sin(c+dx)}}dx - \frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^2+4b\sin(c+dx)a+b^2}{\sqrt{a+b\sin(c+dx)}}dx-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(4a\int \sqrt{a+b\sin(c+dx)}dx-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(4a\int \sqrt{a+b\sin(c+dx)}dx-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{3}\left(\frac{4a\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4a\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

3.52. $\int (a+b\sin(c+dx))^{3/2}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}\right)-\frac{2b\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

input
```
Int[(a + b*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*b*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*d) + ((8*a*EllipticE[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b*Sin
[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]]))
/3
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(217) = 434$.

Time = 0.58 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.93

method	result
default	$\frac{2a^3\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,F\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)+\frac{2\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\mathrm{s}}{}}}{3}}{}$

input
```
int((a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*a^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))+((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2*a-((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3-4*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^3+4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a*b^2+sin(d*x+c)^3*b^3+sin(d*x+c)^2*a*b^2-sin(d*x+c)*b^3-
a*b^2)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

3.52.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.42

$$\int (a+b\sin(c+dx))^{3/2}\,dx = \frac{-12i\sqrt{2}a\sqrt{i\,b}b\text{weierstrassZeta}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}\right.\right.}{}$$

input
```
integrate((a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(-12*I*sqrt(2)*a*sqrt(I*b)*b*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d
*x + c) - 2*I*a)/b)) + 12*I*sqrt(2)*a*sqrt(-I*b)*b*weierstrassZeta(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-
4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 6*sqrt(b*sin(d*x + c) + a)*b^2*
cos(d*x + c) + sqrt(2)*(a^2 + 3*b^2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
- 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(a^2 + 3*b^2)*sqrt(-I*b)*weiers
trassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3,
1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b))/(b*d)
```

3.52.6 Sympy [F]

$$\int (a+b\sin(c+dx))^{3/2}\,dx = \int \left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sin(c + d*x))**(3/2), x)
```

3.52.7 Maxima [F]

$$\int (a+b\sin(c+dx))^{3/2}\,dx = \int \left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(3/2), x)
```

3.52.8 Giac [F]

$$\int (a+b\sin(c+dx))^{3/2}\,dx = \int \left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(3/2), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^{3/2}\,dx=\int {\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((a + b*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*sin(c + d*x))^(3/2), x)
```

3.53 $\int \sqrt{a+b\sin(c+dx)}\,dx$

3.53.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \sqrt{a+b\sin(c+dx)}\,dx = \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\,|\,\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

output
```
-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)
/d/((a+b*sin(d*x+c))/(a+b))^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\sin(c+dx)}\,dx = -\frac{2E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

input
```
Integrate[Sqrt[a + b*Sin[c + d*x]],x]
```

output
```
(-2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x
]])/(d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

3.53.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{2\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

input
```
Int[Sqrt[a + b*Sin[c + d*x]],x]
```

output
```
(2*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/
(d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

3.53.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(91) = 182$.

Time = 0.91 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.85

method	result
default	$\frac{2(a-b)\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\left(F\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a+F\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)b-E\left(\sqrt{\right.\right.}{b\cos(dx+c)\sqrt{a+b\sin(dx+c)}\,d}$
risch	Expression too large to display

input
```
int((a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a-b)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)/b*(EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*b-EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a-EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b)/cos(d*x
+c)/(a+b*sin(d*x+c))^(1/2)/d
```

3.53.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 363, normalized size of antiderivative = 5.85

$$\int \sqrt{a + b\sin(c + dx)}\,dx$$
$$= \frac{\sqrt{2}a\sqrt{i\,b}\text{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3}, \frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3\,b}\right) + \sqrt{2}a\sqrt{-i\,b}\text{weiers}}{}$$

input
```
integrate((a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*a*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) -
 2*I*a)/b) + sqrt(2)*a*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(
d*x + c) + 2*I*a)/b) - 3*I*sqrt(2)*sqrt(I*b)*b*weierstrassZeta(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 3*I*sqrt(2)*sqrt(-I*b)*b*weierstrassZ
eta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstras
sPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)))/(b*d)
```

3.53.6 Sympy [F]

$$\int \sqrt{a + b\sin(c + dx)}\,dx = \int \sqrt{a + b\sin{(c + dx)}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(c + d*x)), x)
```

3.53.7 Maxima [F]

$$\int \sqrt{a+b\sin(c+dx)}\,dx=\int \sqrt{b\sin(dx+c)+a}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(d*x + c) + a), x)
```

3.53.8 Giac [F]

$$\int \sqrt{a+b\sin(c+dx)}\,dx=\int \sqrt{b\sin(dx+c)+a}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(d*x + c) + a), x)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \sqrt{a+b\sin(c+dx)}\,dx=\frac{2\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}{d\,\sqrt{\frac{a+b\,\sin\left(c+d\,x\right)}{a+b}}}$$

input
```
int((a + b*sin(c + d*x))^(1/2),x)
```

output
```
(2*ellipticE(c/2 - pi/4 + (d*x)/2, (2*b)/(a + b))*(a + b*sin(c + d*x))^(1/
2))/(d*((a + b*sin(c + d*x))/(a + b))^(1/2))
```

3.54 $\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx$

3.54.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{d\sqrt{a+b\sin(c+dx)}}$$

output
```
-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx=-\frac{2\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{d\sqrt{a+b\sin(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sin[c + d*x]],x]
```

output
```
(-2*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*
x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

3.54.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx$$

$$\Big\downarrow \text{ 3142}$$

$$\frac{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}$$

input
```
Int[1/Sqrt[a + b*Sin[c + d*x]],x]
```

output
```
(2*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

3.54.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.54.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.03

method	result	size
default	$\frac{2(a-b)\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,F\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)}{b\cos(dx+c)\sqrt{a+b\sin(dx+c)}\,d}$	126

input
```
int(1/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a-b)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

3.54.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.39

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\sqrt{i\,b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3\,b}\right)+\sqrt{2}\sqrt{-i\,b}\,\mathrm{weierstra}}{bd}$$

input
```
integrate(1/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a
)/b) + sqrt(2)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c)
 + 2*I*a)/b))/(b*d)
```

3.54.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sin{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sin(c + d*x)), x)
```

3.54.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sin(d*x + c) + a), x)
```

3.54.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin(dx+c)+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sin(d*x + c) + a), x)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|\,\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\sin(c+d\,x)}{a+b}}}{d\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}$$

input
```
int(1/(a + b*sin(c + d*x))^(1/2),x)
```

output
```
-(2*ellipticF(pi/4 - c/2 - (d*x)/2, (2*b)/(a + b))*((a + b*sin(c + d*x))/(
a + b))^(1/2))/(d*(a + b*sin(c + d*x))^(1/2))
```

3.55 $\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx$

3.55.1 Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\cos(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

output
```
2*b*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*sin(d*
x+c))/(a+b))^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\cos(c+dx)-2(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{(a-b)(a+b)d\sqrt{a+b\sin(c+dx)}}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(-3/2),x]
```

output
```
(2*b*Cos[c + d*x] - 2*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/((a - b)*(a + b)*d*Sqrt[a + b*Sin[
c + d*x]])
```

3.55.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{2b\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}-\frac{2\int -\frac{1}{2}\sqrt{a+b\sin(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{2b\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{2b\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2b\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2b\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}$$

$\Big\downarrow$ 3132

$$\frac{2b\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}+\frac{2\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

input
```
Int[(a + b*Sin[c + d*x])^(-3/2),x]
```

output
```
(2*b*Cos[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (2*EllipticE
[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/((a^2 - b^2)
*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(138) = 276$.

Time = 0.37 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.99

method	result
default	$\frac{2a^2\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,F\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)-2\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1-}{}}}{}$

input
```
int(1/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))-((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*b^2-((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2+((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-sin(d*x+c)^2*b^2+
b^2)/(a^2-b^2)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

3.55.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.38

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{6\sqrt{b\sin(dx+c)+a}\,ab^2\cos(dx+c)+\left(\sqrt{2}ab\sin(dx+c)+\sqrt{2}a^2\right)\sqrt{i\,b}\,\text{weierst}}{}$$

input
```
integrate(1/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*sqrt(b*sin(d*x + c) + a)*b^2*cos(d*x + c) + (sqrt(2)*a*b*sin(d*x +
c) + sqrt(2)*a^2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c
) - 2*I*a)/b) + (sqrt(2)*a*b*sin(d*x + c) + sqrt(2)*a^2)*sqrt(-I*b)*weiers
trassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3,
1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*(-I*sqrt(2)*b^2
*sin(d*x + c) - I*sqrt(2)*a*b)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) - 2*I*a)/b)) + 3*(I*sqrt(2)*b^2*sin(d*x + c) + I*sqrt(2)*a
*b)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 +
 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I
*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)
/b))/((a^2*b^2 - b^4)*d*sin(d*x + c) + (a^3*b - a*b^3)*d)
```

3.55.6 Sympy [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sin{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sin(c + d*x))**(-3/2), x)
```

3.55.7 Maxima [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sin{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(-3/2), x)
```

3.55.8 Giac [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(-3/2), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b*sin(c + d*x))^(3/2), x)
```

3.56 $\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx$

3.56.1 Optimal result

Integrand size = 14, antiderivative size = 231

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2b\cos(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\cos(c+dx)}{3(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} + \frac{8aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3(a^2-b^2)^2\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} - \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{3(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}}$$

output
```
2/3*b*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)+8/3*a*b*cos(d*x+c)/(a^
2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)-8/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/((a+b*sin(d*x+c))/(a+b
))^(1/2)+2/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*
x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2\left(-4a(a+b)^2E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}+(a-b)(a+b)^2\right.}{3(a-b)^2(}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(-5/2),x]
```

output
```
(2*(-4*a*(a + b)^2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b
*Sin[c + d*x])/(a + b))^(3/2) + (a - b)*(a + b)^2*EllipticF[(-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + b*Cos[c + d
*x]*(5*a^2 - b^2 + 4*a*b*Sin[c + d*x])))/(3*(a - b)^2*(a + b)^2*d*(a + b*S
in[c + d*x])^(3/2))
```

3.56.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{2b\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}-\frac{2\int -\frac{3a-b\sin(c+dx)}{2(a+b\sin(c+dx))^{3/2}}dx}{3(a^2-b^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a-b\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}dx}{3(a^2-b^2)}+\frac{2b\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-b\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3233}$$

$$\frac{\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}-\frac{2\int -\frac{3a^2+4b\sin(c+dx)a+b^2}{2\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{3a^2+4b\sin(c+dx)a+b^2}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{3a^2+4b\sin(c+dx)a+b^2}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3231}$$

$$\frac{\frac{4a\int \sqrt{a+b\sin(c+dx)}dx-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{4a\int \sqrt{a+b\sin(c+dx)}dx-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{\frac{\frac{4a\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

3.56. $\int \frac{1}{(a+b\sin(c+dx))^{5/2}}dx$

$$\frac{\frac{\frac{4a\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{2b\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\frac{\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{2b\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{2b\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}}{a^2-b^2}+\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{2b\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2b\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}+\frac{\frac{8ab\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}+\frac{\frac{8a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}}{a^2-b^2}}{3(a^2-b^2)}$$

input
```
Int[(a + b*Sin[c + d*x])^(-5/2),x]
```

3.56. $\int\frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx$

output
```
(2*b*Cos[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + ((8*a*b*
Cos[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + ((8*a*EllipticE[(
c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x
]]))/(a^2 - b^2))/(3*(a^2 - b^2))
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.56.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.15

method	result
default	$\frac{\sqrt{-(-b\sin(dx+c)-a)(\cos^2(dx+c))}\left(\frac{2\sqrt{-(-b\sin(dx+c)-a)\left(\cos^2(dx+c)\right)}}{3b\left(a^2-b^2\right)\left(\sin(dx+c)+\frac{a}{b}\right)^2}+\frac{8b\left(\cos^2(dx+c)\right)a}{3\left(a^2-b^2\right)^2\sqrt{-(-b\sin(dx+c)-a)\left(\cos^2(dx+c)\right)}}+\frac{2\left(3a^2+b^2\right)\left(\frac{a}{b}\right.}{}\right.}{}$

input
```
int(1/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(2/3/b/(a^2-b^2)*(-(-b*sin(d*x+c)-
a)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+c)^2/(a^2-b^2)^2*a
/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3*a^4-6*a^2*b^2+3*
b^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)
*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b/(a^2-
b^2)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/
2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)
*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))/cos(d*x+c)/
(a+b*sin(d*x+c))^(1/2)/d
```

3.56.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.16

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}(a^2b^2+3\,b^4)\cos\left(dx+c\right)^2-2\,\sqrt{2}(a^3b+3\,ab^3)\sin\left(dx+c\right)-\sqrt{2}(a^4+4\,a\right.}{}$$

input
```
integrate(1/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*((sqrt(2)*(a^2*b^2 + 3*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(a^3*b + 3*a*b^
3)*sin(d*x + c) - sqrt(2)*(a^4 + 4*a^2*b^2 + 3*b^4))*sqrt(I*b)*weierstrass
PInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3
*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + (sqrt(2)*(a^2*b^2 + 3*b
^4)*cos(d*x + c)^2 - 2*sqrt(2)*(a^3*b + 3*a*b^3)*sin(d*x + c) - sqrt(2)*(a
^4 + 4*a^2*b^2 + 3*b^4))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*si
n(d*x + c) + 2*I*a)/b) + 12*(-I*sqrt(2)*a*b^3*cos(d*x + c)^2 + 2*I*sqrt(2)
*a^2*b^2*sin(d*x + c) + sqrt(2)*(I*a^3*b + I*a*b^3))*sqrt(I*b)*weierstrass
Zeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstras
sPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 12*(I*sqrt(2)*a*b^3*c
os(d*x + c)^2 - 2*I*sqrt(2)*a^2*b^2*sin(d*x + c) + sqrt(2)*(-I*a^3*b - I*a
*b^3))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^
3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-
8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I
*a)/b)) - 6*(4*a*b^3*cos(d*x + c)*sin(d*x + c) + (5*a^2*b^2 - b^4)*cos(d*x
 + c))*sqrt(b*sin(d*x + c) + a))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x +
c)^2 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*sin(d*x + c) - (a^6*b - a^4*b^3 -
 a^2*b^5 + b^7)*d)
```

3.56.6 Sympy [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sin(c + d*x))**(-5/2), x)
```

3.56.7 Maxima [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sin\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(-5/2), x)
```

3.56.8 Giac [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sin\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(-5/2), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+b\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b*sin(c + d*x))^(5/2), x)
```

3.57 $\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx$

3.57.1 Optimal result

Integrand size = 14, antiderivative size = 292

$$\begin{aligned}\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx &= \frac{2b\cos(c+dx)}{5(a^2-b^2)\,d(a+b\sin(c+dx))^{5/2}} \\ &+ \frac{16ab\cos(c+dx)}{15(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} + \frac{2b(23a^2+9b^2)\cos(c+dx)}{15(a^2-b^2)^3\,d\sqrt{a+b\sin(c+dx)}} \\ &+ \frac{2(23a^2+9b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{15(a^2-b^2)^3\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} \\ &- \frac{16a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{15(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}}\end{aligned}$$

output
```
2/5*b*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(5/2)+16/15*a*b*cos(d*x+c)/(
a^2-b^2)^2/d/(a+b*sin(d*x+c))^(3/2)+2/15*b*(23*a^2+9*b^2)*cos(d*x+c)/(a^2-
b^2)^3/d/(a+b*sin(d*x+c))^(1/2)-2/15*(23*a^2+9*b^2)*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/((a+b*sin(
d*x+c))/(a+b))^(1/2)+16/15*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/
2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx = \frac{2\left(-\frac{\left(\left(23a^2+9b^2\right)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)+8a(-a+b)\operatorname{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),\frac{2b}{a+b}\right)\right)\left(\frac{a+b\sin(c+d}{a+b}\right.}{(a-b)^3}\right.}{15d(a+b\sin(}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(-7/2),x]
```

output
```
(2*(-((((23*a^2 + 9*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] +
 8*a*(-a + b)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin
[c + d*x])/(a + b))^(5/2))/(a - b)^3) + (b*Cos[c + d*x]*(34*a^4 - 5*a^2*b^
2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Sin[c + d*x] + b^2*(23*a^2 + 9*b^2)*Sin
[c + d*x]^2))/(a^2 - b^2)^3))/(15*d*(a + b*Sin[c + d*x])^(5/2))
```

3.57.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}-\frac{2\int -\frac{5a-3b\sin(c+dx)}{2(a+b\sin(c+dx))^{5/2}}dx}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-3b\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}dx}{5\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-3b\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}dx}{5(a^2-b^2)}+\frac{2b\cos(c+dx)}{5d(a^2-b^2)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}-\frac{2\int -\frac{3(5a^2+3b^2)-8ab\sin(c+dx)}{2(a+b\sin(c+dx))^{3/2}}dx}{3(a^2-b^2)}}{5(a^2-b^2)}+\frac{2b\cos(c+dx)}{5d(a^2-b^2)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3(5a^2+3b^2)-8ab\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}dx}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5(a^2-b^2)}+\frac{2b\cos(c+dx)}{5d(a^2-b^2)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3(5a^2+3b^2)-8ab\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}dx}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5(a^2-b^2)}+\frac{2b\cos(c+dx)}{5d(a^2-b^2)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\frac{\frac{2b(23a^2+9b^2)\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}-\frac{2\int -\frac{a(15a^2+17b^2)+b(23a^2+9b^2)\sin(c+dx)}{2\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5(a^2-b^2)}+\frac{2b\cos(c+dx)}{5d(a^2-b^2)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{a(15a^2+17b^2)+b(23a^2+9b^2)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{2b(23a^2+9b^2)\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5(a^2-b^2)}+\frac{2b\cos(c+dx)}{5d(a^2-b^2)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{a(15a^2+17b^2)+b(23a^2+9b^2)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{2b(23a^2+9b^2)\cos(c+dx)}{d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5(a^2-b^2)}+\frac{2b\cos(c+dx)}{5d(a^2-b^2)(a+b\sin(c+dx))^{5/2}}$$

3.57. $\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \; 3231$$

$$\frac{\frac{\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\sin(c+dx)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\sin(c+dx)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3134$$

$$\frac{\frac{\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3132$$

$$\frac{\frac{\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}+\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}+\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}$$

3.57. $\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx$

$\big\downarrow$ 3142

$$\frac{\frac{\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}}{a^2-b^2}+\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}$$
$$\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\sin(c+dx)\right)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}dx}{\sqrt{a+b\sin(c+dx)}}}{a^2-b^2}+\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}}{3(a^2-b^2)}+\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}$$
$$\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\sin(c+dx)\right)^{5/2}}$$

$\big\downarrow$ 3140

$$\frac{2b\cos(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\sin(c+dx)\right)^{5/2}}+$$
$$\frac{\frac{16ab\cos(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}+\frac{\frac{2b\left(23a^2+9b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

input
```
Int[(a + b*Sin[c + d*x])^(-7/2),x]
```

output
```
(2*b*Cos[c + d*x])/(5*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(5/2)) + ((16*a*b
*Cos[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + ((2*b*(23*a^
2 + 9*b^2)*Cos[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + ((2*(2
3*a^2 + 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin
[c + d*x]])/(d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Ell
ipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b
)])/(d*Sqrt[a + b*Sin[c + d*x]]))/(a^2 - b^2))/(3*(a^2 - b^2)))/(5*(a^2 -
b^2))
```

3.57. $\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx$

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.57.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.00

method	result
default	$\sqrt{-(-b\sin(dx+c)-a)(\cos^2(dx+c))}\left(\frac{2\sqrt{-(-b\sin(dx+c)-a)\left(\cos^2(dx+c)\right)}}{5b^2\left(a^2-b^2\right)\left(\sin(dx+c)+\frac{a}{b}\right)^3}+\frac{16a\sqrt{-(-b\sin(dx+c)-a)\left(\cos^2(dx+c)\right)}}{15\left(a^2-b^2\right)^2b\left(\sin(dx+c)+\frac{a}{b}\right)^2}+\frac{2b\left(\cos^2(dx+\right.}{15\left(a^2-b^2\right)^3\sqrt{-(-b\,\mathrm{si}}}\right.$

input
```
int(1/(a+b*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(2/5/b^2/(a^2-b^2)*(-(-b*sin(d*x+c
)-a)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^3+16/15*a/(a^2-b^2)^2/b*(-(-b*si
n(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+2/15*b*cos(d*x+c)^2/(a^
2-b^2)^3*(23*a^2+9*b^2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*(15*a^3+
17*a*b^2)/(15*a^6-45*a^4*b^2+45*a^2*b^4-15*b^6)*(a/b-1)*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2
)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))+2/15*b*(23*a^2+9*b^2)/(a^2-b^2)^3*(a/b-1)*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c
))*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))/cos(d*x+c)/(a+b*sin(d*x+c))^
(1/2)/d
```

3.57.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.60

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/45*((3*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cos(d*x + c)^2 + (sqrt(2)*(a^3*b^
3 - 33*a*b^5)*cos(d*x + c)^2 - sqrt(2)*(3*a^5*b - 98*a^3*b^3 - 33*a*b^5))*
sin(d*x + c) - sqrt(2)*(a^6 - 30*a^4*b^2 - 99*a^2*b^4))*sqrt(I*b)*weierstr
assPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3
*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + (3*sqrt(2)*(a^4*b^2
- 33*a^2*b^4)*cos(d*x + c)^2 + (sqrt(2)*(a^3*b^3 - 33*a*b^5)*cos(d*x + c)^
2 - sqrt(2)*(3*a^5*b - 98*a^3*b^3 - 33*a*b^5))*sin(d*x + c) - sqrt(2)*(a^6
 - 30*a^4*b^2 - 99*a^2*b^4))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*
b*sin(d*x + c) + 2*I*a)/b) - 3*(3*sqrt(2)*(-23*I*a^3*b^3 - 9*I*a*b^5)*cos(
d*x + c)^2 + (sqrt(2)*(-23*I*a^2*b^4 - 9*I*b^6)*cos(d*x + c)^2 + sqrt(2)*(
69*I*a^4*b^2 + 50*I*a^2*b^4 + 9*I*b^6))*sin(d*x + c) + sqrt(2)*(23*I*a^5*b
 + 78*I*a^3*b^3 + 27*I*a*b^5))*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) - 2*I*a)/b)) - 3*(3*sqrt(2)*(23*I*a^3*b^3 + 9*I*a*b^5)*cos
(d*x + c)^2 + (sqrt(2)*(23*I*a^2*b^4 + 9*I*b^6)*cos(d*x + c)^2 + sqrt(2)*(
-69*I*a^4*b^2 - 50*I*a^2*b^4 - 9*I*b^6))*sin(d*x + c) + sqrt(2)*(-23*I*a^5
*b - 78*I*a^3*b^3 - 27*I*a*b^5))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*...
```

3.57.6 Sympy [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\sin{(c+dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))**(7/2),x)
```

output
```
Integral((a + b*sin(c + d*x))**(-7/2), x)
```

3.57.7 Maxima [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(-7/2), x)
```

3.57.8 Giac [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(-7/2), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\sin(c+d\,x))^{7/2}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x))^(7/2),x)
```

output
```
int(1/(a + b*sin(c + d*x))^(7/2), x)
```

3.58 $\int (a+b\sin(c+dx))^{4/3}\,dx$

3.58.1 Optimal result

Integrand size = 14, antiderivative size = 109

$$\int (a+b\sin(c+dx))^{4/3}\,dx = -\frac{\sqrt{2}(a+b)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)\sqrt[3]{a+b\sin(c+dx)}}{d\sqrt{1+\sin(c+dx)}\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}$$

output
```
-(a+b)*AppellF1(1/2,-4/3,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c)
)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/3)*2^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/
3)/(1+sin(d*x+c))^(1/2)
```

3.58.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(109) = 218.

Time = 1.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.24

$$\int (a+b\sin(c+dx))^{4/3}\,dx = -\frac{3\sec(c+dx)\sqrt[3]{a+b\sin(c+dx)}\left(4b^2\cos^2(c+dx)+4(a^2-b^2)\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+d}{a+b}\right.\right.}{}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(4/3),x]
```

output
```
(-3*Sec[c + d*x]*(a + b*Sin[c + d*x])^(1/3)*(4*b^2*Cos[c + d*x]^2 + 4*(a^2
 - b^2)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(b*(1
 + Sin[c + d*x]))/(-a + b)] - 5*a*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Sin[
c + d*x])/(a - b), (a + b*Sin[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Sin[c + d
*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])
))/(16*b*d)
```

3.58.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx))^{4/3}dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{(a+b\sin(c+dx))^{4/3}}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{(a+b)\cos(c+dx)\sqrt[3]{a+b\sin(c+dx)}\int \frac{\left(\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}\right)^{4/3}}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2}(a+b)\cos(c+dx)\sqrt[3]{a+b\sin(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}$$

input
```
Int[(a + b*Sin[c + d*x])^(4/3),x]
```

output
```
-((Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sin[c + d*x])/2, (b*
(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1/3))/(d*S
qrt[1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^(1/3)))
```

3.58.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.58.4 Maple [F]

$$\int (a+b\sin (dx+c))^{\frac{4}{3}}dx$$

input
```
int((a+b*sin(d*x+c))^(4/3),x)
```

output
```
int((a+b*sin(d*x+c))^(4/3),x)
```

3.58.5 Fricas [F]

$$\int (a+b\sin(c+dx))^{4/3}\,dx = \int (b\sin (dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c) + a)^(4/3), x)
```

3.58.6 Sympy [F]

$$\int (a+b\sin(c+dx))^{4/3}\,dx = \int (a+b\sin (c+dx))^{\frac{4}{3}}\;dx$$

input
```
integrate((a+b*sin(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sin(c + d*x))**(4/3), x)
```

3.58.7 Maxima [F]

$$\int (a+b\sin(c+dx))^{4/3}\,dx = \int (b\sin(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(4/3), x)
```

3.58.8 Giac [F]

$$\int (a+b\sin(c+dx))^{4/3}\,dx = \int (b\sin(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(4/3), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^{4/3}\,dx = \int (a+b\,\sin(c+d\,x))^{4/3}\,dx$$

input
```
int((a + b*sin(c + d*x))^(4/3),x)
```

output
```
int((a + b*sin(c + d*x))^(4/3), x)
```

3.59 $\int (a+b\sin(c+dx))^{2/3}\,dx$

3.59.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int (a+b\sin(c+dx))^{2/3}\,dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+b\sin(c+dx))^{2/3}}{d\sqrt{1+\sin(c+dx)}\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{2/3}}$$

output
```
-AppellF1(1/2,-2/3,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(
d*x+c)*(a+b*sin(d*x+c))^(2/3)*2^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(2/3)/(1+
sin(d*x+c))^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11

$$\int (a+b\sin(c+dx))^{2/3}\,dx = \frac{3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}(a}{5bd}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(2/3),x]
```

output
```
(3*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[c
 + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(5/3))/(5*b*d)
```

3.59.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx))^{2/3} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\sin(c+dx))^{2/3}dx$$

$$\Big\downarrow \text{ 3144}$$

$$\frac{\cos(c+dx)\int \frac{(a+b\sin(c+dx))^{2/3}}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{ 156}$$

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{ 155}$$

$$-\frac{\sqrt{2}\cos(c+dx)(a+b\sin(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{2/3}}$$

input
```
Int[(a + b*Sin[c + d*x])^(2/3),x]
```

output
```
-((Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin
[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^(2/3))/(d*Sqrt[1 +
Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^(2/3)))
```

3.59.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.59.4 Maple [F]

$$\int (a + b\sin(dx + c))^{\frac{2}{3}}\, dx$$

input
```
int((a+b*sin(d*x+c))^(2/3),x)
```

output
```
int((a+b*sin(d*x+c))^(2/3),x)
```

3.59.5 Fricas [F]

$$\int (a + b\sin(c + dx))^{2/3} \, dx = \int (b\sin(dx + c) + a)^{\frac{2}{3}} \, dx$$

input
```
integrate((a+b*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c) + a)^(2/3), x)
```

3.59.6 Sympy [F]

$$\int (a + b\sin(c + dx))^{2/3} \, dx = \int (a + b\sin(c + dx))^{\frac{2}{3}} \, dx$$

input
```
integrate((a+b*sin(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sin(c + d*x))**(2/3), x)
```

3.59.7 Maxima [F]

$$\int (a + b\sin(c + dx))^{2/3} \, dx = \int (b\sin(dx + c) + a)^{\frac{2}{3}} \, dx$$

input
```
integrate((a+b*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(2/3), x)
```

3.59.8 Giac [F]

$$\int (a+b\sin(c+dx))^{2/3}\,dx = \int (b\sin(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(2/3), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^{2/3}\,dx = \int (a+b\,\sin(c+d\,x))^{2/3}\,dx$$

input
```
int((a + b*sin(c + d*x))^(2/3),x)
```

output
```
int((a + b*sin(c + d*x))^(2/3), x)
```

3.60 $\int \sqrt[3]{a+b\sin(c+dx)}\,dx$

3.60.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \sqrt[3]{a+b\sin(c+dx)}\,dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)\sqrt[3]{a+b\sin(c+dx)}}{d\sqrt{1+\sin(c+dx)}\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}$$

output
```
-AppellF1(1/2,-1/3,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(
d*x+c)*(a+b*sin(d*x+c))^(1/3)*2^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/3)/(1+
sin(d*x+c))^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11

$$\int \sqrt[3]{a+b\sin(c+dx)}\,dx = \frac{3\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}(a+b\sin(c+}{4bd}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(1/3),x]
```

output
```
(3*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[c
 + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(4/3))/(4*b*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{\sqrt[3]{a+b\sin(c+dx)}}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\cos(c+dx)\sqrt[3]{a+b\sin(c+dx)}\int \frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2}\cos(c+dx)\sqrt[3]{a+b\sin(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}$$

input
```
Int[(a + b*Sin[c + d*x])^(1/3),x]
```

output
```
-((Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin
[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1/3))/(d*Sqrt[1 +
Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^(1/3)))
```

3.60.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

3.60.4 Maple [F]

$$\int (a+b\sin(dx+c))^{\frac{1}{3}}\, dx$$

input
```
int((a+b*sin(d*x+c))^(1/3),x)
```

output
```
int((a+b*sin(d*x+c))^(1/3),x)
```

3.60.5 Fricas [F]

$$\int \sqrt[3]{a+b\sin(c+dx)}\, dx = \int (b\sin(dx+c)+a)^{\frac{1}{3}}\, dx$$

input
```
integrate((a+b*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c) + a)^(1/3), x)
```

3.60.6 Sympy [F]

$$\int \sqrt[3]{a+b\sin(c+dx)}\, dx = \int \sqrt[3]{a+b\sin(c+dx)}\, dx$$

input
```
integrate((a+b*sin(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sin(c + d*x))**(1/3), x)
```

3.60.7 Maxima [F]

$$\int \sqrt[3]{a+b\sin(c+dx)}\,dx = \int (b\sin(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(1/3), x)
```

3.60.8 Giac [F]

$$\int \sqrt[3]{a+b\sin(c+dx)}\,dx = \int (b\sin(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*sin(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(1/3), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\sin(c+dx)}\,dx = \int (a+b\,\sin(c+d\,x))^{1/3}\,dx$$

input
```
int((a + b*sin(c + d*x))^(1/3),x)
```

output
```
int((a + b*sin(c + d*x))^(1/3), x)
```

3.61 $\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx$

3.61.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}{d\sqrt{1+\sin(c+dx)}\,\sqrt[3]{a+b\sin(c+dx)}}$$

output
```
-AppellF1(1/2,1/3,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d
*x+c)*((a+b*sin(d*x+c))/(a+b))^(1/3)*2^(1/2)/d/(a+b*sin(d*x+c))^(1/3)/(1+s
in(d*x+c))^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx = \frac{3\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}(a+b\sin(c+}{2bd}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(-1/3),x]
```

output
```
(3*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[c
 + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(2/3))/(2*b*d)
```

3.61.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{1}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\sqrt[3]{a+b\sin(c+dx)}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\cos(c+dx)\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\sqrt[3]{a+b\sin(c+dx)}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2}\cos(c+dx)\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}\sqrt[3]{a+b\sin(c+dx)}}$$

input
```
Int[(a + b*Sin[c + d*x])^(-1/3),x]
```

output
```
-((Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[
c + d*x]))/(a + b)]*Cos[c + d*x]*((a + b*Sin[c + d*x])/(a + b))^(1/3))/(d*
Sqrt[1 + Sin[c + d*x]]*(a + b*Sin[c + d*x])^(1/3)))
```

3.61.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.61.4 Maple [F]

$$\int \frac{1}{\left(a+b\sin\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(a+b*sin(d*x+c))^(1/3),x)
```

output
```
int(1/(a+b*sin(d*x+c))^(1/3),x)
```

3.61.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx=\int \frac{1}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c) + a)^(-1/3), x)
```

3.61.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{a+b\sin\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sin(c + d*x))**(-1/3), x)
```

3.61.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(-1/3), x)
```

3.61.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(-1/3), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+b\sin(c+dx)}}\,dx = \int \frac{1}{(a+b\,\sin(c+d\,x))^{1/3}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x))^(1/3),x)
```

output
```
int(1/(a + b*sin(c + d*x))^(1/3), x)
```

3.62 $\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx$

3.62.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{2/3}}{d\sqrt{1+\sin(c+dx)}(a+b\sin(c+dx))^{2/3}}$$

output
```
-AppellF1(1/2,2/3,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d
*x+c)*((a+b*sin(d*x+c))/(a+b))^(2/3)*2^(1/2)/d/(a+b*sin(d*x+c))^(2/3)/(1+s
in(d*x+c))^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx = \frac{3\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}{bd}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(-2/3),x]
```

output
```
(3*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[c
 + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1/3))/(b*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{1}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}(a+b\sin(c+dx))^{2/3}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\cos(c+dx)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}\right)^{2/3}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}(a+b\sin(c+dx))^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2}\cos(c+dx)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}(a+b\sin(c+dx))^{2/3}}$$

input
```
Int[(a + b*Sin[c + d*x])^(-2/3),x]
```

output
```
-((Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[
c + d*x]))/(a + b)]*Cos[c + d*x]*((a + b*Sin[c + d*x])/(a + b))^(2/3))/(d*
Sqrt[1 + Sin[c + d*x]]*(a + b*Sin[c + d*x])^(2/3)))
```

3.62.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

3.62.4 Maple [F]

$$\int \frac{1}{(a + b\sin(dx + c))^{\frac{2}{3}}} dx$$

input
```
int(1/(a+b*sin(d*x+c))^(2/3),x)
```

output
```
int(1/(a+b*sin(d*x+c))^(2/3),x)
```

3.62.5 Fricas [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c) + a)^(-2/3), x)
```

3.62.6 Sympy [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(a+b\sin(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sin(c + d*x))**(-2/3), x)
```

3.62.7 Maxima [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(-2/3), x)
```

3.62.8 Giac [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(-2/3), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx = \int \frac{1}{(a+b\sin(c+dx))^{2/3}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x))^(2/3),x)
```

output
```
int(1/(a + b*sin(c + d*x))^(2/3), x)
```

3.63 $\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx$

3.63.1 Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}}{(a+b)d\sqrt{1+\sin(c+dx)}\sqrt[3]{a+b\sin(c+dx)}}$$

output
```
-AppellF1(1/2,4/3,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d
*x+c)*((a+b*sin(d*x+c))/(a+b))^(1/3)*2^(1/2)/(a+b)/d/(a+b*sin(d*x+c))^(1/3
)/(1+sin(d*x+c))^(1/2)
```

3.63.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 262 vs. 2(111) = 222.

Time = 1.35 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.36

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx = -\frac{3\sec(c+dx)\left(5a\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}(a+b\,\text{s}}{}$$

input
```
Integrate[(a + b*Sin[c + d*x])^(-4/3),x]
```

output
```
(-3*Sec[c + d*x]*(5*a*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Sin[c + d*x])/(a
 - b), (a + b*Sin[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b
))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))]*(a + b*Sin[c + d*x]) - 2*(5*b^
2*Cos[c + d*x]^2 + 2*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Sin[c + d*x])/(a
- b), (a + b*Sin[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b)
)]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^2)))/(10*b*(
a^2 - b^2)*d*(a + b*Sin[c + d*x])^(1/3))
```

3.63.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}dx$$
$$\Big\downarrow \text{ 3144}$$
$$\frac{\cos(c+dx)\int \frac{1}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}(a+b\sin(c+dx))^{4/3}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$
$$\Big\downarrow \text{ 156}$$
$$\frac{\cos(c+dx)\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}\right)^{4/3}}d\sin(c+dx)}{d(a+b)\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}\sqrt[3]{a+b\sin(c+dx)}}$$
$$\Big\downarrow \text{ 155}$$
$$-\frac{\sqrt{2}\cos(c+dx)\sqrt[3]{\frac{a+b\sin(c+dx)}{a+b}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sin(c+dx)+1}\sqrt[3]{a+b\sin(c+dx)}}$$

input
```
Int[(a + b*Sin[c + d*x])^(-4/3),x]
```

output
```
-((Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[
c + d*x]))/(a + b)]*Cos[c + d*x]*((a + b*Sin[c + d*x])/(a + b))^(1/3))/((a
 + b)*d*Sqrt[1 + Sin[c + d*x]]*(a + b*Sin[c + d*x])^(1/3)))
```

3.63.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

3.63.4 Maple [F]

$$\int \frac{1}{(a+b\sin(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(1/(a+b*sin(d*x+c))^(4/3),x)
```

output
```
int(1/(a+b*sin(d*x+c))^(4/3),x)
```

3.63.5 Fricas [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(d*x + c) + a)^(2/3)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2), x)
```

3.63.6 Sympy [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+b\sin(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sin(c + d*x))**(-4/3), x)
```

3.63.7 Maxima [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^(-4/3), x)
```

3.63.8 Giac [F]

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^(-4/3), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+b\,\sin(c+d\,x))^{4/3}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x))^(4/3),x)
```

output
```
int(1/(a + b*sin(c + d*x))^(4/3), x)
```

3.64 $\int (a+b\sin(c+dx))^n\,dx$

3.64.1 Optimal result

Integrand size = 12, antiderivative size = 104

$$\int (a+b\sin(c+dx))^n\,dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+d}{a+b}\right.}{d\sqrt{1+\sin(c+dx)}}$$

output
```
-AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*
x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/d/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*x+
c))^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15

$$\int (a+b\sin(c+dx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}(a+b\,s}{bd(1+n)}$$

input
```
Integrate[(a + b*Sin[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin
[c + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*
Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(
1 + n))
```

3.64.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx))^n dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{(a+b\sin(c+dx))^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}\int \frac{\left(\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}\right)^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2}\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}}$$

input
```
Int[(a + b*Sin[c + d*x])^n,x]
```

output
```
-((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
 + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(d*Sqrt[1 + Sin[c
+ d*x]]*((a + b*Sin[c + d*x])/(a + b))^n))
```

3.64.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

3.64.4 Maple [F]

$$\int (a + b\sin(dx + c))^n \, dx$$

input
```
int((a+b*sin(d*x+c))^n,x)
```

output
```
int((a+b*sin(d*x+c))^n,x)
```

3.64.5 Fricas [F]

$$\int (a+b\sin(c+dx))^n\,dx = \int (b\sin(dx+c)+a)^n\,dx$$

input
```
integrate((a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c) + a)^n, x)
```

3.64.6 Sympy [F]

$$\int (a+b\sin(c+dx))^n\,dx = \int (a+b\sin(c+dx))^n\,dx$$

input
```
integrate((a+b*sin(d*x+c))**n,x)
```

output
```
Integral((a + b*sin(c + d*x))**n, x)
```

3.64.7 Maxima [F]

$$\int (a+b\sin(c+dx))^n\,dx = \int (b\sin(dx+c)+a)^n\,dx$$

input
```
integrate((a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c) + a)^n, x)
```

3.64.8 Giac [F]

$$\int (a+b\sin(c+dx))^n\,dx = \int (b\sin(dx+c)+a)^n\,dx$$

input
```
integrate((a+b*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c) + a)^n, x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^n\,dx = \int (a+b\,\sin(c+d\,x))^n\,dx$$

input
```
int((a + b*sin(c + d*x))^n,x)
```

output
```
int((a + b*sin(c + d*x))^n, x)
```

3.65 $\int (3+4\sin(c+dx))^n\,dx$

3.65.1 Optimal result

Integrand size = 12, antiderivative size = 72

$$\int (3+4\sin(c+dx))^n\,dx = -\frac{\sqrt{2}7^n\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{4}{7}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt{1+\sin(c+dx)}}$$

output
```
-7^n*AppellF1(1/2,1/2,-n,3/2,1/2-1/2*sin(d*x+c),4/7-4/7*sin(d*x+c))*cos(d*
x+c)*2^(1/2)/d/(1+sin(d*x+c))^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int (3+4\sin(c+dx))^n\,dx = \frac{\mathrm{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,-3-4\sin(c+dx),\frac{1}{7}(3+4\sin(c+dx))\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)(3+4\sin}{\sqrt{7}d(1+n)}$$

input
```
Integrate[(3 + 4*Sin[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 1/2, 1/2, 2 + n, -3 - 4*Sin[c + d*x], (3 + 4*Sin[c + d*x]
)/7]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*(3 + 4*Sin[c + d*x])^(1 + n))/(Sqrt
[7]*d*(1 + n))
```

3.65.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3144, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4\sin(c+dx)+3)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (4\sin(c+dx)+3)^n dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{(4\sin(c+dx)+3)^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2}7^n\cos(c+dx)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),\frac{4}{7}(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

input
```
Int[(3 + 4*Sin[c + d*x])^n,x]
```

output
```
-((Sqrt[2]*7^n*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (4*(1 - S
in[c + d*x]))/7]*Cos[c + d*x])/(d*Sqrt[1 + Sin[c + d*x]]))
```

3.65.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.65.4 Maple [F]

$$\int (3+4\sin (dx+c))^n\,dx$$

input
```
int((3+4*sin(d*x+c))^n,x)
```

output
```
int((3+4*sin(d*x+c))^n,x)
```

3.65.5 Fricas [F]

$$\int (3+4\sin(c+dx))^n\,dx = \int (4\,\sin (dx+c)+3)^n\,dx$$

input
```
integrate((3+4*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((4*sin(d*x + c) + 3)^n, x)
```

3.65.6 Sympy [F]

$$\int (3+4\sin(c+dx))^n\,dx = \int (4\sin (c+dx)+3)^n\,dx$$

input
```
integrate((3+4*sin(d*x+c))**n,x)
```

output
```
Integral((4*sin(c + d*x) + 3)**n, x)
```

3.65.7 Maxima [F]

$$\int (3+4\sin(c+dx))^n\,dx = \int (4\,\sin{(dx+c)}+3)^n\,dx$$

input
```
integrate((3+4*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((4*sin(d*x + c) + 3)^n, x)
```

3.65.8 Giac [F]

$$\int (3+4\sin(c+dx))^n\,dx = \int (4\,\sin{(dx+c)}+3)^n\,dx$$

input
```
integrate((3+4*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((4*sin(d*x + c) + 3)^n, x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int (3+4\sin(c+dx))^n\,dx = \int (4\,\sin{(c+d\,x)}+3)^n\,dx$$

input
```
int((4*sin(c + d*x) + 3)^n,x)
```

output
```
int((4*sin(c + d*x) + 3)^n, x)
```

3.66 $\int (3-4\sin(c+dx))^n \, dx$

3.66.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int (3-4\sin(c+dx))^n \, dx = \frac{\sqrt{2}7^n \operatorname{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2},\frac{3}{2},\frac{4}{7}(1+\sin(c+dx)),\frac{1}{2}(1+\sin(c+dx))\right)\cos(c+dx)}{d\sqrt{1-\sin(c+dx)}}$$

output
```
7^n*AppellF1(1/2,1/2,-n,3/2,1/2+1/2*sin(d*x+c),4/7+4/7*sin(d*x+c))*cos(d*x
+c)*2^(1/2)/d/(1-sin(d*x+c))^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int (3-4\sin(c+dx))^n \, dx = -\frac{\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{1}{7}(3-4\sin(c+dx)),-3+4\sin(c+dx)\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)(3-4}{\sqrt{7}d(1+n)}$$

input
```
Integrate[(3 - 4*Sin[c + d*x])^n,x]
```

output
```
-((AppellF1[1 + n, 1/2, 1/2, 2 + n, (3 - 4*Sin[c + d*x])/7, -3 + 4*Sin[c +
 d*x]]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*(3 - 4*Sin[c + d*x])^(1 + n))/(Sq
rt[7]*d*(1 + n)))
```

3.66.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3144, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3-4\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (3-4\sin(c+dx))^n dx$$

$$\Big\downarrow \text{ 3144}$$

$$\frac{\cos(c+dx)\int \frac{(3-4\sin(c+dx))^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}7^n\cos(c+dx)\operatorname{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2},\frac{3}{2},\frac{4}{7}(\sin(c+dx)+1),\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

input
```
Int[(3 - 4*Sin[c + d*x])^n,x]
```

output
```
(Sqrt[2]*7^n*AppellF1[1/2, -n, 1/2, 3/2, (4*(1 + Sin[c + d*x]))/7, (1 + Si
n[c + d*x])/2]*Cos[c + d*x])/(d*Sqrt[1 - Sin[c + d*x]])
```

3.66.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.66.4 Maple [F]

$$\int (3-4\sin(dx+c))^n\, dx$$

input
```
int((3-4*sin(d*x+c))^n,x)
```

output
```
int((3-4*sin(d*x+c))^n,x)
```

3.66.5 Fricas [F]

$$\int (3-4\sin(c+dx))^n\, dx = \int (-4\, \sin(dx+c)+3)^n\, dx$$

input
```
integrate((3-4*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((-4*sin(d*x + c) + 3)^n, x)
```

3.66.6 Sympy [F]

$$\int (3-4\sin(c+dx))^n\, dx = \int (3-4\sin(c+dx))^n\ dx$$

input
```
integrate((3-4*sin(d*x+c))**n,x)
```

output
```
Integral((3 - 4*sin(c + d*x))**n, x)
```

3.66.7 Maxima [F]

$$\int (3-4\sin(c+dx))^n\,dx = \int (-4\sin(dx+c)+3)^n\,dx$$

input
```
integrate((3-4*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((-4*sin(d*x + c) + 3)^n, x)
```

3.66.8 Giac [F]

$$\int (3-4\sin(c+dx))^n\,dx = \int (-4\sin(dx+c)+3)^n\,dx$$

input
```
integrate((3-4*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((-4*sin(d*x + c) + 3)^n, x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int (3-4\sin(c+dx))^n\,dx = \int (3-4\sin(c+d\,x))^n\,dx$$

input
```
int((3 - 4*sin(c + d*x))^n,x)
```

output
```
int((3 - 4*sin(c + d*x))^n, x)
```

3.67 $\int (4+3\sin(c+dx))^n\,dx$

3.67.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int (4+3\sin(c+dx))^n\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1+\sin(c+dx)),-3(1+\sin(c+dx))\right)\cos(c+dx)}{d\sqrt{1-\sin(c+dx)}}$$

output
```
AppellF1(1/2,-n,1/2,3/2,-3-3*sin(d*x+c),1/2+1/2*sin(d*x+c))*cos(d*x+c)*2^(
1/2)/d/(1-sin(d*x+c))^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.55

$$\int (4+3\sin(c+dx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,4+3\sin(c+dx),\frac{1}{7}(4+3\sin(c+dx))\right)\sec(c+dx)\sqrt{-1-\sin(c+dx)}\sqrt{1-}}{\sqrt{7}d(1+n)}$$

input
```
Integrate[(4 + 3*Sin[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 1/2, 1/2, 2 + n, 4 + 3*Sin[c + d*x], (4 + 3*Sin[c + d*x])
/7]*Sec[c + d*x]*Sqrt[-1 - Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]*(4 + 3*Sin
[c + d*x])^(1 + n))/(Sqrt[7]*d*(1 + n))
```

3.67.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3144, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(c+dx)+4)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (3\sin(c+dx)+4)^n dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{(3\sin(c+dx)+4)^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\cos(c+dx)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(\sin(c+dx)+1),-3(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

input
```
Int[(4 + 3*Sin[c + d*x])^n,x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 + Sin[c + d*x])/2, -3*(1 + Sin[c +
 d*x])]*Cos[c + d*x])/(d*Sqrt[1 - Sin[c + d*x]])
```

3.67.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.67.4 Maple [F]

$$\int (4+3\sin(dx+c))^n\,dx$$

input
```
int((4+3*sin(d*x+c))^n,x)
```

output
```
int((4+3*sin(d*x+c))^n,x)
```

3.67.5 Fricas [F]

$$\int (4+3\sin(c+dx))^n\,dx=\int (3\sin(dx+c)+4)^n\,dx$$

input
```
integrate((4+3*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((3*sin(d*x + c) + 4)^n, x)
```

3.67.6 Sympy [F]

$$\int (4+3\sin(c+dx))^n\,dx=\int (3\sin(c+dx)+4)^n\,dx$$

input
```
integrate((4+3*sin(d*x+c))**n,x)
```

output
```
Integral((3*sin(c + d*x) + 4)**n, x)
```

3.67.7 Maxima [F]

$$\int (4+3\sin(c+dx))^n\,dx = \int (3\,\sin(dx+c)+4)^n\,dx$$

input
```
integrate((4+3*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((3*sin(d*x + c) + 4)^n, x)
```

3.67.8 Giac [F]

$$\int (4+3\sin(c+dx))^n\,dx = \int (3\,\sin(dx+c)+4)^n\,dx$$

input
```
integrate((4+3*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((3*sin(d*x + c) + 4)^n, x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int (4+3\sin(c+dx))^n\,dx = \int (3\,\sin(c+d\,x)+4)^n\,dx$$

input
```
int((3*sin(c + d*x) + 4)^n,x)
```

output
```
int((3*sin(c + d*x) + 4)^n, x)
```

3.68 $\int (4 - 3\sin(c + dx))^n \, dx$

3.68.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int (4 - 3\sin(c + dx))^n \, dx = \frac{\sqrt{2}7^n \operatorname{AppellF1}\left(\frac{1}{2}, -n, \frac{1}{2}, \frac{3}{2}, \frac{3}{7}(1 + \sin(c + dx)), \frac{1}{2}(1 + \sin(c + dx))\right) \cos(c + dx)}{d\sqrt{1 - \sin(c + dx)}}$$

output
```
7^n*AppellF1(1/2,1/2,-n,3/2,1/2+1/2*sin(d*x+c),3/7+3/7*sin(d*x+c))*cos(d*x
+c)*2^(1/2)/d/(1-sin(d*x+c))^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.39

$$\int (4 - 3\sin(c + dx))^n \, dx = -\frac{\operatorname{AppellF1}\left(1 + n, \frac{1}{2}, \frac{1}{2}, 2 + n, \frac{1}{7}(4 - 3\sin(c + dx)), 4 - 3\sin(c + dx)\right) \sec(c + dx)(4 - 3\sin(c + dx))^{1+n}}{\sqrt{7}d(1 + n)}$$

input
```
Integrate[(4 - 3*Sin[c + d*x])^n,x]
```

output
```
-((AppellF1[1 + n, 1/2, 1/2, 2 + n, (4 - 3*Sin[c + d*x])/7, 4 - 3*Sin[c +
d*x]]*Sec[c + d*x]*(4 - 3*Sin[c + d*x])^(1 + n)*Sqrt[-1 + Sin[c + d*x]]*Sq
rt[1 + Sin[c + d*x]])/(Sqrt[7]*d*(1 + n)))
```

3.68.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3144, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4-3\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (4-3\sin(c+dx))^n dx$$

$$\Big\downarrow \text{ 3144}$$

$$\frac{\cos(c+dx)\int \frac{(4-3\sin(c+dx))^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}7^n\cos(c+dx)\operatorname{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2},\frac{3}{2},\frac{3}{7}(\sin(c+dx)+1),\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

input
```
Int[(4 - 3*Sin[c + d*x])^n,x]
```

output
```
(Sqrt[2]*7^n*AppellF1[1/2, -n, 1/2, 3/2, (3*(1 + Sin[c + d*x]))/7, (1 + Si
n[c + d*x])/2]*Cos[c + d*x])/(d*Sqrt[1 - Sin[c + d*x]])
```

3.68.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.68.4 Maple [F]

$$\int (4-3\sin (dx+c))^{n}\, dx$$

input
```
int((4-3*sin(d*x+c))^n,x)
```

output
```
int((4-3*sin(d*x+c))^n,x)
```

3.68.5 Fricas [F]

$$\int (4-3\sin(c+dx))^n\, dx = \int (-3\, \sin (dx+c)+4)^{n}\, dx$$

input
```
integrate((4-3*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((-3*sin(d*x + c) + 4)^n, x)
```

3.68.6 Sympy [F]

$$\int (4-3\sin(c+dx))^n\, dx = \int (4-3\sin (c+dx))^{n}\; dx$$

input
```
integrate((4-3*sin(d*x+c))**n,x)
```

output
```
Integral((4 - 3*sin(c + d*x))**n, x)
```

3.68.7 Maxima [F]

$$\int (4-3\sin(c+dx))^n\,dx = \int (-3\,\sin(dx+c)+4)^n\,dx$$

input
```
integrate((4-3*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((-3*sin(d*x + c) + 4)^n, x)
```

3.68.8 Giac [F]

$$\int (4-3\sin(c+dx))^n\,dx = \int (-3\,\sin(dx+c)+4)^n\,dx$$

input
```
integrate((4-3*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((-3*sin(d*x + c) + 4)^n, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int (4-3\sin(c+dx))^n\,dx = \int (4-3\,\sin(c+d\,x))^n\,dx$$

input
```
int((4 - 3*sin(c + d*x))^n,x)
```

output
```
int((4 - 3*sin(c + d*x))^n, x)
```

3.69 $\int (-3+4\sin(c+dx))^n \, dx$

3.69.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int (-3+4\sin(c+dx))^n \, dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),4(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt{1+\sin(c+dx)}}$$

output
```
-AppellF1(1/2,-n,1/2,3/2,4-4*sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)*2^(
1/2)/d/(1+sin(d*x+c))^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (-3+4\sin(c+dx))^n \, dx = \frac{\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{1}{7}(3-4\sin(c+dx)),-3+4\sin(c+dx)\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)(-3+4}{\sqrt{7}d(1+n)}$$

input
```
Integrate[(-3 + 4*Sin[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 1/2, 1/2, 2 + n, (3 - 4*Sin[c + d*x])/7, -3 + 4*Sin[c + d
*x]]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*(-3 + 4*Sin[c + d*x])^(1 + n))/(Sqr
t[7]*d*(1 + n))
```

3.69.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3144, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4\sin(c+dx)-3)^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (4\sin(c+dx)-3)^n dx$$

$$\Big\downarrow \text{ 3144}$$

$$\frac{\cos(c+dx)\int \frac{(4\sin(c+dx)-3)^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{ 155}$$

$$-\frac{\sqrt{2}\cos(c+dx)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(c+dx)),4(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

input
```
Int[(-3 + 4*Sin[c + d*x])^n,x]
```

output
```
-((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, 4*(1 - Sin[c
+ d*x])]*Cos[c + d*x])/(d*Sqrt[1 + Sin[c + d*x]]))
```

3.69.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.69.4 Maple [F]

$$\int (-3 + 4\sin(dx+c))^n \, dx$$

input
```
int((-3+4*sin(d*x+c))^n,x)
```

output
```
int((-3+4*sin(d*x+c))^n,x)
```

3.69.5 Fricas [F]

$$\int (-3 + 4\sin(c+dx))^n \, dx = \int (4 \sin(dx+c) - 3)^n \, dx$$

input
```
integrate((-3+4*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((4*sin(d*x + c) - 3)^n, x)
```

3.69.6 Sympy [F]

$$\int (-3 + 4\sin(c+dx))^n \, dx = \int (4\sin(c+dx) - 3)^n \, dx$$

input
```
integrate((-3+4*sin(d*x+c))**n,x)
```

output
```
Integral((4*sin(c + d*x) - 3)**n, x)
```

3.69.7 Maxima [F]

$$\int (-3+4\sin(c+dx))^n\,dx = \int (4\,\sin(dx+c)-3)^n\,dx$$

input
```
integrate((-3+4*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((4*sin(d*x + c) - 3)^n, x)
```

3.69.8 Giac [F]

$$\int (-3+4\sin(c+dx))^n\,dx = \int (4\,\sin(dx+c)-3)^n\,dx$$

input
```
integrate((-3+4*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((4*sin(d*x + c) - 3)^n, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int (-3+4\sin(c+dx))^n\,dx = \int (4\,\sin(c+d\,x)-3)^n\,dx$$

input
```
int((4*sin(c + d*x) - 3)^n,x)
```

output
```
int((4*sin(c + d*x) - 3)^n, x)
```

3.70 $\int(-3-4\sin(c+dx))^n\,dx$

3.70.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int(-3-4\sin(c+dx))^n\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2},\frac{3}{2},4(1+\sin(c+dx)),\frac{1}{2}(1+\sin(c+dx))\right)\cos(c+dx)}{d\sqrt{1-\sin(c+dx)}}$$

output
```
AppellF1(1/2,-n,1/2,3/2,4+4*sin(d*x+c),1/2+1/2*sin(d*x+c))*cos(d*x+c)*2^(1
/2)/d/(1-sin(d*x+c))^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

$$\int(-3-4\sin(c+dx))^n\,dx = -\frac{\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,-3-4\sin(c+dx),\frac{1}{7}(3+4\sin(c+dx))\right)\sqrt{\cos^2(c+dx)}\sec(c+dx)(-3-}{\sqrt{7}d(1+n)}$$

input
```
Integrate[(-3 - 4*Sin[c + d*x])^n,x]
```

output
```
-((AppellF1[1 + n, 1/2, 1/2, 2 + n, -3 - 4*Sin[c + d*x], (3 + 4*Sin[c + d*
x])/7]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*(-3 - 4*Sin[c + d*x])^(1 + n))/(S
qrt[7]*d*(1 + n)))
```

3.70.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3144, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-4\sin(c+dx)-3)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-4\sin(c+dx)-3)^n dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{(-4\sin(c+dx)-3)^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\cos(c+dx)\,\text{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2},\frac{3}{2},4(\sin(c+dx)+1),\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

input
```
Int[(-3 - 4*Sin[c + d*x])^n,x]
```

output
```
(Sqrt[2]*AppellF1[1/2, -n, 1/2, 3/2, 4*(1 + Sin[c + d*x]), (1 + Sin[c + d*
x])/2]*Cos[c + d*x])/(d*Sqrt[1 - Sin[c + d*x]])
```

3.70.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.70.4 Maple [F]

$$\int (-3-4\sin(dx+c))^n\,dx$$

input
```
int((-3-4*sin(d*x+c))^n,x)
```

output
```
int((-3-4*sin(d*x+c))^n,x)
```

3.70.5 Fricas [F]

$$\int (-3-4\sin(c+dx))^n\,dx=\int (-4\,\sin(dx+c)-3)^n\,dx$$

input
```
integrate((-3-4*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((-4*sin(d*x + c) - 3)^n, x)
```

3.70.6 Sympy [F]

$$\int (-3-4\sin(c+dx))^n\,dx=\int (-4\sin(c+dx)-3)^n\,dx$$

input
```
integrate((-3-4*sin(d*x+c))**n,x)
```

output
```
Integral((-4*sin(c + d*x) - 3)**n, x)
```

3.70.7 Maxima [F]

$$\int (-3-4\sin(c+dx))^n\,dx = \int (-4\,\sin\left(dx+c\right)-3)^n\,dx$$

input
```
integrate((-3-4*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((-4*sin(d*x + c) - 3)^n, x)
```

3.70.8 Giac [F]

$$\int (-3-4\sin(c+dx))^n\,dx = \int (-4\,\sin\left(dx+c\right)-3)^n\,dx$$

input
```
integrate((-3-4*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((-4*sin(d*x + c) - 3)^n, x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int (-3-4\sin(c+dx))^n\,dx = \int (-4\,\sin\left(c+d\,x\right)-3)^n\,dx$$

input
```
int((- 4*sin(c + d*x) - 3)^n,x)
```

output
```
int((- 4*sin(c + d*x) - 3)^n, x)
```

3.71 $\int (-4+3\sin(c+dx))^n\,dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 95

$$\int (-4+3\sin(c+dx))^n\,dx = \frac{\sqrt{2}7^n \operatorname{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2},\frac{3}{2},\frac{3}{7}(1+\sin(c+dx)),\frac{1}{2}(1+\sin(c+dx))\right)\cos(c+dx)(4-3\sin(c+dx))^{-n}(-4$$
$$d\sqrt{1-\sin(c+dx)}$$

output
```
7^n*AppellF1(1/2,1/2,-n,3/2,1/2+1/2*sin(d*x+c),3/7+3/7*sin(d*x+c))*cos(d*x
+c)*(-4+3*sin(d*x+c))^n*2^(1/2)/d/((4-3*sin(d*x+c))^n)/(1-sin(d*x+c))^(1/2
)
```

3.71.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int (-4+3\sin(c+dx))^n\,dx = \frac{\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{1}{7}(4-3\sin(c+dx)),4-3\sin(c+dx)\right)\sec(c+dx)\sqrt{-1+\sin(c+dx)}\sqrt{1+}$$
$$\sqrt{7}d(1+n)$$

input
```
Integrate[(-4 + 3*Sin[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 1/2, 1/2, 2 + n, (4 - 3*Sin[c + d*x])/7, 4 - 3*Sin[c + d*
x]]*Sec[c + d*x]*Sqrt[-1 + Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]]*(-4 + 3*Si
n[c + d*x])^(1 + n))/(Sqrt[7]*d*(1 + n))
```

3.71.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(c+dx)-4)^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (3\sin(c+dx)-4)^n dx$$

$$\Big\downarrow \ 3144$$

$$\frac{\cos(c+dx)\int \frac{(3\sin(c+dx)-4)^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \ 156$$

$$\frac{\cos(c+dx)(4-3\sin(c+dx))^{-n}(3\sin(c+dx)-4)^n\int \frac{(4-3\sin(c+dx))^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \ 155$$

$$\frac{\sqrt{2}7^n\cos(c+dx)(4-3\sin(c+dx))^{-n}(3\sin(c+dx)-4)^n\operatorname{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2},\frac{3}{2},\frac{3}{7}(\sin(c+dx)+1),\frac{1}{2}(\sin(c+dx}{d\sqrt{1-\sin(c+dx)}}$$

input
```
Int[(-4 + 3*Sin[c + d*x])^n,x]
```

output
```
(Sqrt[2]*7^n*AppellF1[1/2, -n, 1/2, 3/2, (3*(1 + Sin[c + d*x]))/7, (1 + Si
n[c + d*x])/2]*Cos[c + d*x]*(-4 + 3*Sin[c + d*x])^n)/(d*(4 - 3*Sin[c + d*x
])^n*Sqrt[1 - Sin[c + d*x]])
```

3.71.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

3.71.4 Maple [F]

$$\int (-4+3\sin(dx+c))^n \, dx$$

input
```
int((-4+3*sin(d*x+c))^n,x)
```

output
```
int((-4+3*sin(d*x+c))^n,x)
```

3.71.5 Fricas [F]

$$\int (-4+3\sin(c+dx))^n \, dx = \int (3 \sin(dx+c)-4)^n \, dx$$

input
```
integrate((-4+3*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((3*sin(d*x + c) - 4)^n, x)
```

3.71.6 Sympy [F]

$$\int (-4+3\sin(c+dx))^n \, dx = \int (3\sin(c+dx)-4)^n \, dx$$

input
```
integrate((-4+3*sin(d*x+c))**n,x)
```

output
```
Integral((3*sin(c + d*x) - 4)**n, x)
```

3.71.7 Maxima [F]

$$\int (-4+3\sin(c+dx))^n \, dx = \int (3 \sin(dx+c)-4)^n \, dx$$

input
```
integrate((-4+3*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((3*sin(d*x + c) - 4)^n, x)
```

3.71.8 Giac [F]

$$\int (-4+3\sin(c+dx))^n \, dx = \int (3\,\sin\left(dx+c\right)-4)^n \, dx$$

input
```
integrate((-4+3*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((3*sin(d*x + c) - 4)^n, x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int (-4+3\sin(c+dx))^n \, dx = \int (3\,\sin\left(c+d\,x\right)-4)^n \, dx$$

input
```
int((3*sin(c + d*x) - 4)^n,x)
```

output
```
int((3*sin(c + d*x) - 4)^n, x)
```

3.72 $\int (-4-3\sin(c+dx))^n\,dx$

3.72.1 Optimal result

Integrand size = 12, antiderivative size = 110

$$\int (-4-3\sin(c+dx))^n\,dx = -\frac{\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,4+3\sin(c+dx),\frac{1}{7}(4+3\sin(c+dx))\right)\cos(c+dx)(-4-3\sin(c+dx))^{1+}}{\sqrt{7}d(1+n)\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))}$$

output
```
-1/7*AppellF1(1+n,1/2,1/2,2+n,4+3*sin(d*x+c),4/7+3/7*sin(d*x+c))*cos(d*x+c
)*(-4-3*sin(d*x+c))^(1+n)*(-1-sin(d*x+c))^(1/2)/d/(1+n)/(1+sin(d*x+c))*7^(
1/2)/(1-sin(d*x+c))^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int (-4-3\sin(c+dx))^n\,dx = -\frac{\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,4+3\sin(c+dx),\frac{1}{7}(4+3\sin(c+dx))\right)\sec(c+dx)(-4-3\sin(c+dx))^{1+}}{\sqrt{7}d(1+n)}$$

input
```
Integrate[(-4 - 3*Sin[c + d*x])^n,x]
```

output
```
-((AppellF1[1 + n, 1/2, 1/2, 2 + n, 4 + 3*Sin[c + d*x], (4 + 3*Sin[c + d*x
])/7]*Sec[c + d*x]*(-4 - 3*Sin[c + d*x])^(1 + n)*Sqrt[-1 - Sin[c + d*x]]*S
qrt[1 - Sin[c + d*x]])/(Sqrt[7]*d*(1 + n)))
```

3.72.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3144, 156, 27, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-3\sin(c+dx)-4)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-3\sin(c+dx)-4)^n dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(c+dx)\int \frac{(-3\sin(c+dx)-4)^n}{\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}\sqrt{\sin(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\sqrt{3}\sqrt{-\sin(c+dx)-1}\cos(c+dx)\int \frac{(-3\sin(c+dx)-4)^n}{\sqrt{3}\sqrt{-\sin(c+dx)-1}\sqrt{1-\sin(c+dx)}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}(\sin(c+dx)+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{-\sin(c+dx)-1}\cos(c+dx)\int \frac{(-3\sin(c+dx)-4)^n}{\sqrt{-\sin(c+dx)-1}\sqrt{1-\sin(c+dx)}}d\sin(c+dx)}{d\sqrt{1-\sin(c+dx)}(\sin(c+dx)+1)}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{-\sin(c+dx)-1}\cos(c+dx)(-3\sin(c+dx)-4)^{n+1}\operatorname{AppellF1}\left(n+1,\frac{1}{2},\frac{1}{2},n+2,3\sin(c+dx)+4,\frac{1}{7}(3\sin(c$$
$$\sqrt{7}d(n+1)\sqrt{1-\sin(c+dx)}(\sin(c+dx)+1)$$

input
```
Int[(-4 - 3*Sin[c + d*x])^n,x]
```

output
```
-((AppellF1[1 + n, 1/2, 1/2, 2 + n, 4 + 3*Sin[c + d*x], (4 + 3*Sin[c + d*x
])/7]*Cos[c + d*x]*(-4 - 3*Sin[c + d*x])^(1 + n)*Sqrt[-1 - Sin[c + d*x]])/
(Sqrt[7]*d*(1 + n)*Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])))
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

3.72.4 Maple [F]

$$\int (-4-3\sin(dx+c))^n\,dx$$

input
```
int((-4-3*sin(d*x+c))^n,x)
```

output
```
int((-4-3*sin(d*x+c))^n,x)
```

3.72.5 Fricas [F]

$$\int (-4-3\sin(c+dx))^n\,dx = \int (-3\sin(dx+c)-4)^n\,dx$$

input
```
integrate((-4-3*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((-3*sin(d*x + c) - 4)^n, x)
```

3.72.6 Sympy [F]

$$\int (-4-3\sin(c+dx))^n\,dx = \int (-3\sin(c+dx)-4)^n\,dx$$

input
```
integrate((-4-3*sin(d*x+c))**n,x)
```

output
```
Integral((-3*sin(c + d*x) - 4)**n, x)
```

3.72.7 Maxima [F]

$$\int (-4-3\sin(c+dx))^n\,dx = \int (-3\,\sin\left(dx+c\right)-4)^n\,dx$$

input
```
integrate((-4-3*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((-3*sin(d*x + c) - 4)^n, x)
```

3.72.8 Giac [F]

$$\int (-4-3\sin(c+dx))^n\,dx = \int (-3\,\sin\left(dx+c\right)-4)^n\,dx$$

input
```
integrate((-4-3*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((-3*sin(d*x + c) - 4)^n, x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int (-4-3\sin(c+dx))^n\,dx = \int (-3\,\sin\left(c+d\,x\right)-4)^n\,dx$$

input
```
int((- 3*sin(c + d*x) - 4)^n,x)
```

output
```
int((- 3*sin(c + d*x) - 4)^n, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```